Using non-convex approximations for efficient analysis of timed automata

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Joint work with F. Herbreteau[1], D. Kini[2] and I. Walukiewicz[1]

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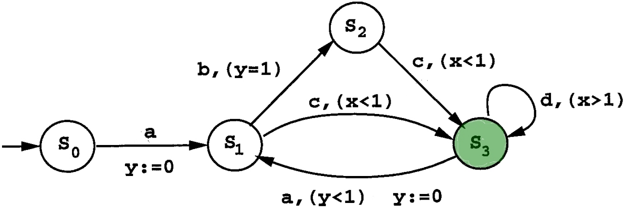
Verification Seminar

Université Libre de Bruxelles

Timed Automata [AD94]



Run: finite **sequence** of transitions,

$$(s_0, \overbrace{0}^{x}, \overbrace{0}^{y}) \xrightarrow{0.4,a} (s_1, 0.4, 0) \xrightarrow{0.5,c} (s_3, 0.9, 0.5)$$

- A run is **accepting** if it ends in a green state.

The problem we are interested in ...

Given a TA, does there **exist** an **accepting run**?

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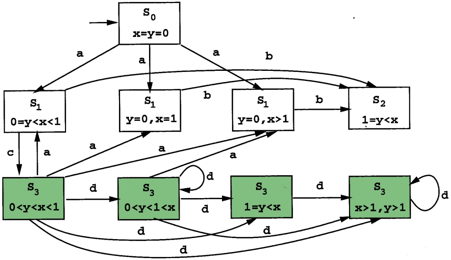
Theorem [AD94, CY92]

This problem is **PSPACE-complete**

First solution to this problem

Key idea: Partition the space of valuations into a **finite** number of **regions**



- Region: set of valuations satisfying the **same guards** w.r.t. time
- Finiteness: Parametrized by **maximal constant**

Sound and complete [AD94]

Region graph preserves state **reachability**

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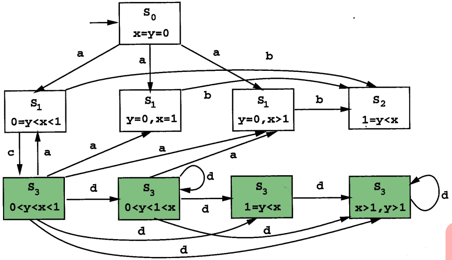

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- Finiteness: Parametrized by **maximal constant**

$\mathcal{O}(|X|!.M^{|X|})$ many regions!

Sound and complete [AD94]

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A more efficient solution...

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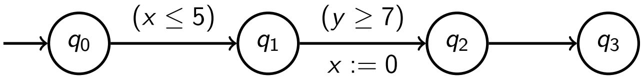

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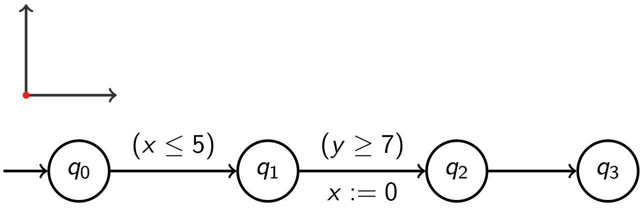

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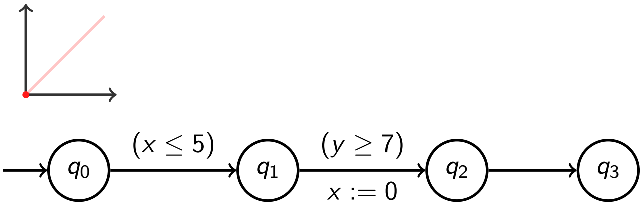

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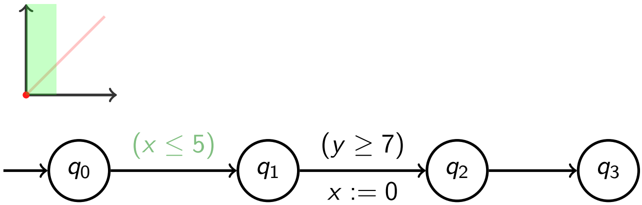

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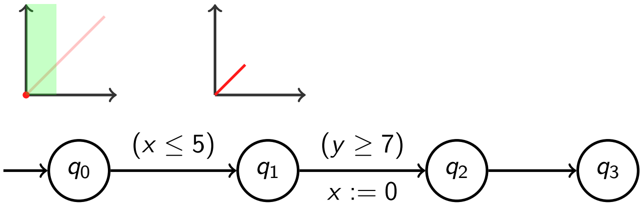

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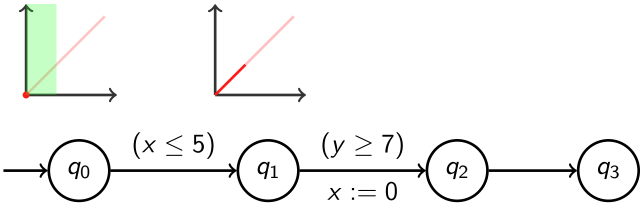

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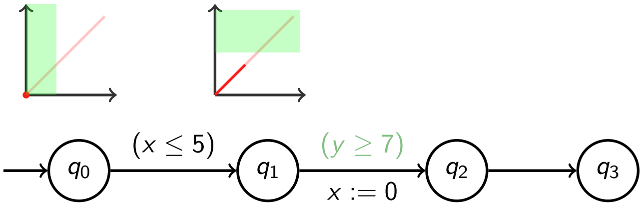

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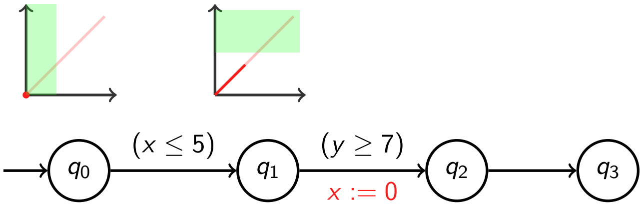

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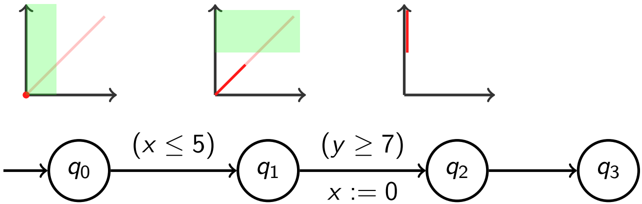

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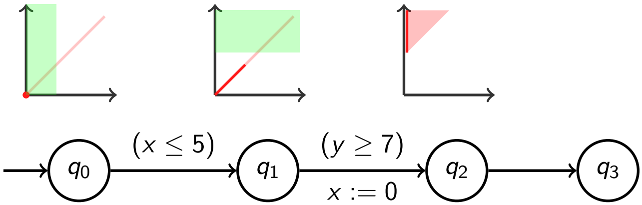

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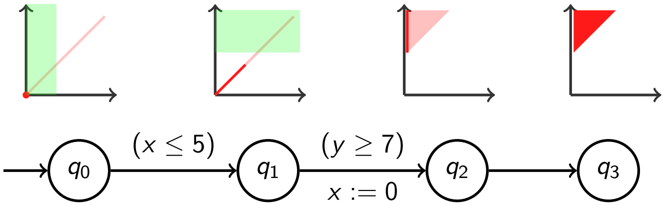

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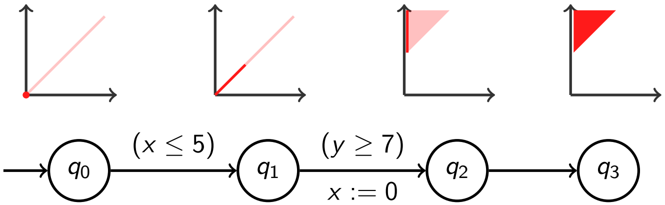

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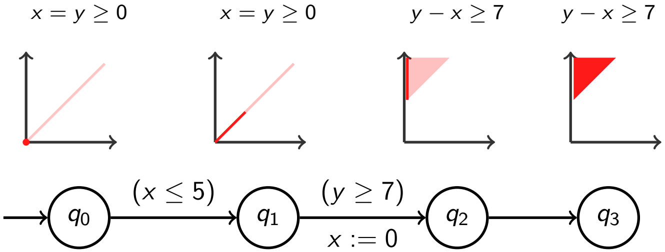

Zones and zone graph

- Zone: set of valuations defined by conjunctions of constraints:
  - $x \sim c$
  - $x - y \sim c$
  - e.g. $(x - y \geq 1) \wedge y < 2$
- Representation: by DBM

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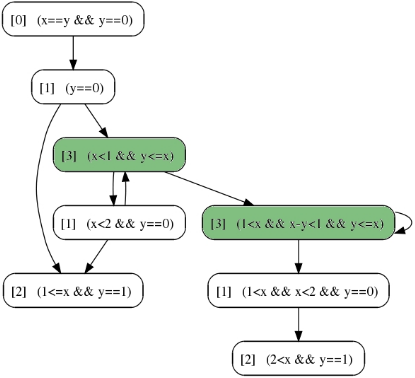

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Sound and complete [DT98]

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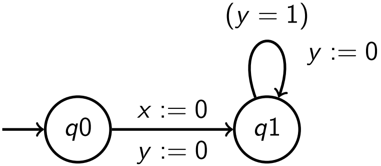
But the zone graph could be infinite ...



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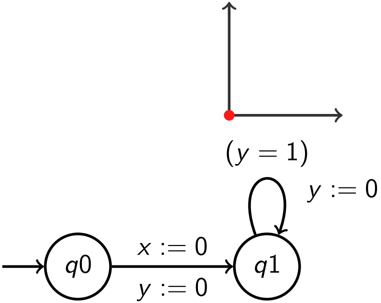

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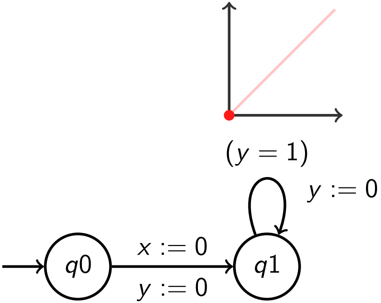

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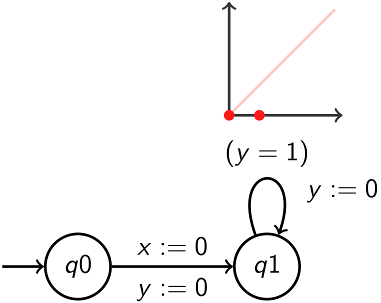

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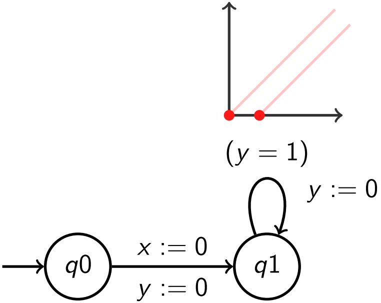

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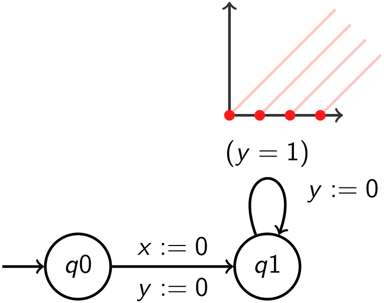

Use finite abstractions

Key idea: **Abstract** each zone in a **sound** manner

$$\begin{tikzcd}
& (q_0, Z_0) \arrow[dl, "/" marking] \arrow[dr] & & \\
{} & & (q_1, Z_1) \arrow[dl, "/" marking] \arrow[dr] & \\
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$$(q_0, \mathfrak{a}(Z_0))$$

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$$\begin{tikzcd} & {(q_0, Z_0)} \arrow[dl, "\times" marking] \arrow[dr] & & \\ {} & & {(q_1, Z_1)} \arrow[dl, "\times" marking] \arrow[dr] & \\ & {} & & {(q_2, Z_2)} \end{tikzcd}$$

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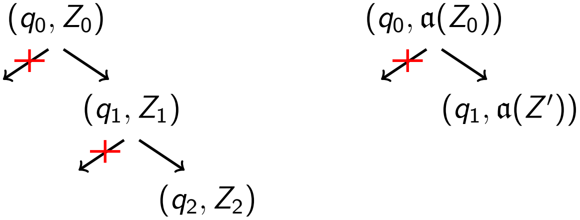

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 & {} & & (q_2, \mathfrak{a}(Z''))
\end{tikzcd}
$$

Use finite abstractions

Key idea: **Abstract** each zone in a **sound** manner

$$(q_0, Z_0) \qquad (q_0, \mathfrak{a}(Z_0))$$

$$(q_1, Z_1) \qquad (q_1, \mathfrak{a}(Z'))$$

$$(q_2, Z_2) \qquad (q_2, \mathfrak{a}(Z''))$$

- Number of **abstracted zones** is **finite**
- **Coarser** abstraction $\rightarrow$ fewer **abstracted zones**

Abstractions in literature [Bou04, BBLP06]

$$\begin{tikzcd}
& \mathfrak{a}_{\preccurlyeq LU} & \\
\mathsf{Closure}_{\alpha} \arrow[ur] & & \mathsf{Extra}^{+}_{LU} \arrow[ul] \\
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& \mathsf{Extra}_{\alpha} \arrow[u] &
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Sound and complete

All the above abstractions preserve state **reachability**

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All the above abstractions preserve state **reachability**

But for **implementation** abstracted zone should be a zone

Abstractions in literature [Bou04, BBLP06]

$$
\begin{tikzcd}
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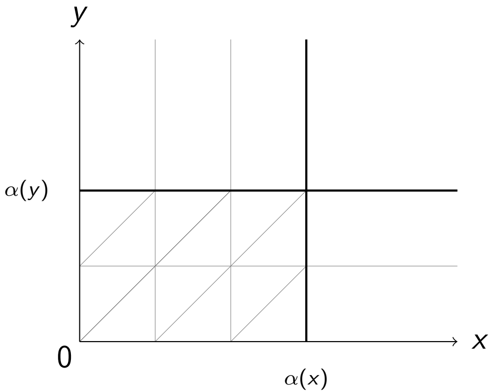
\mathrm{Closure}_{\alpha} \arrow[ur] & & \mathrm{Extra}^{+}_{LU}\ \checkmark \arrow[ul] \\
 & \mathrm{Extra}^{+}_{\alpha}\ \checkmark \arrow[ul] \arrow[ur] & \\
 & \mathrm{Extra}_{\alpha}\ \checkmark \arrow[u] &
\end{tikzcd}
$$

Only convex abstractions in **implementations**!

Here...

Efficient use of the **non-convex** Closure abstraction!

What is Closure$_\alpha$?



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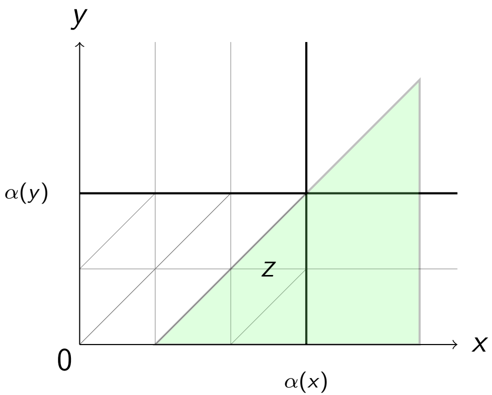

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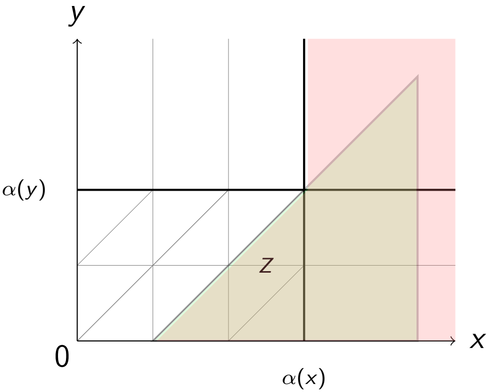
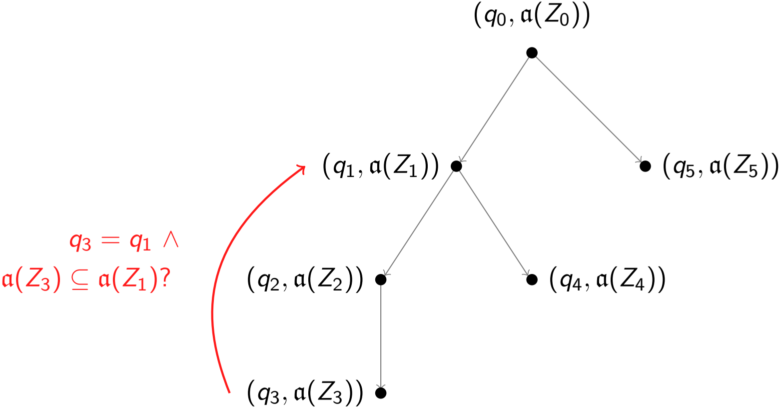


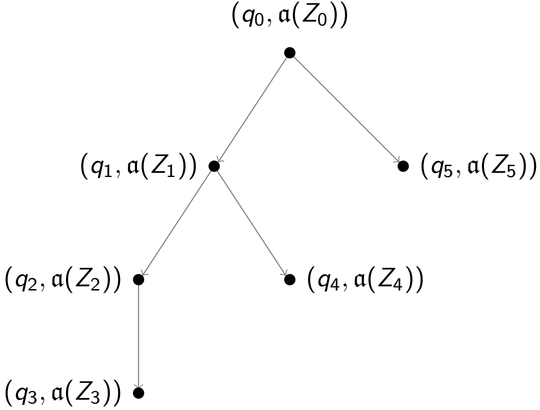
Closure$_\alpha(Z)$: set of regions that Z intersects

Using Closure$_\alpha$ for reachability



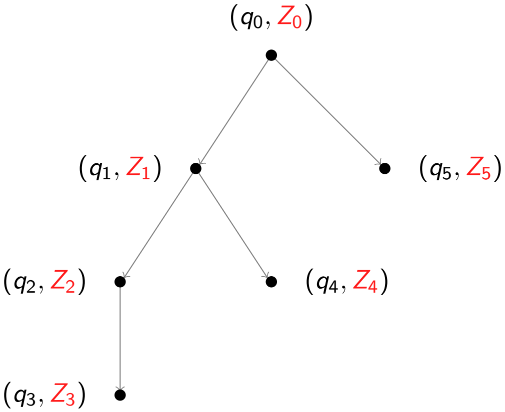
Standard algorithm: **covering tree**

Using Closure$_\alpha$ for reachability



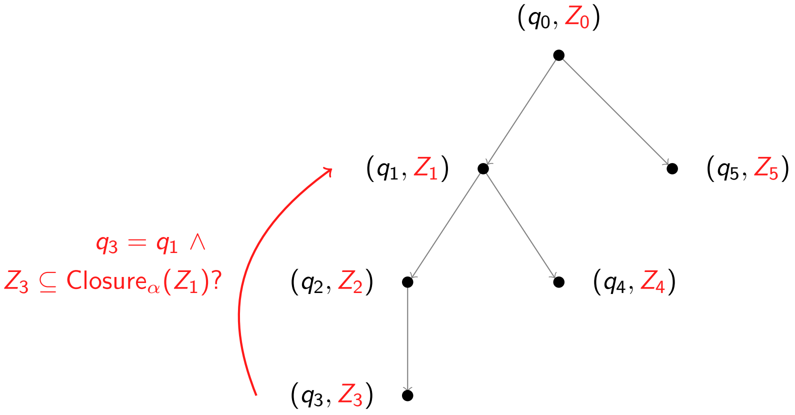
Closure$_\alpha(Z)$ **cannot be** efficiently **stored**

Using Closure$_\alpha$ for reachability



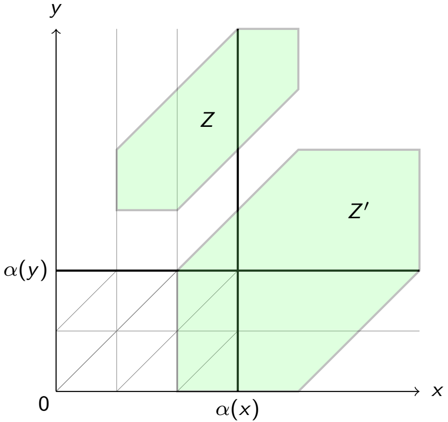
Do **not store** abstracted zones!

Using Closure$_\alpha$ for reachability



Use Closure for termination!

$Z \subseteq \text{Closure}_\alpha(Z')$?



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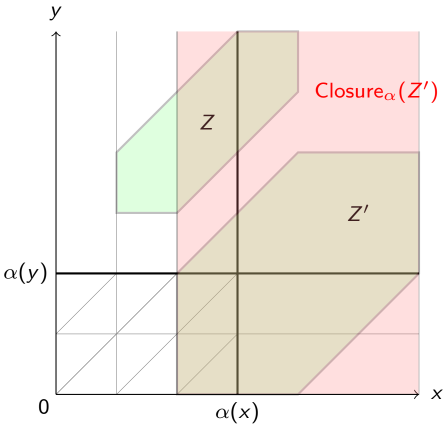

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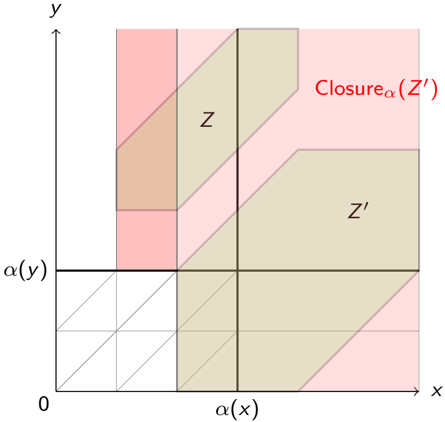

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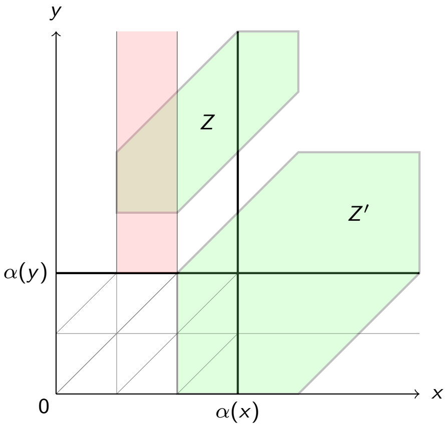

$Z \not\subseteq \mathrm{Closure}_\alpha(Z') \Leftrightarrow \exists R.$ R **intersects Z**, R does **not intersect Z′**

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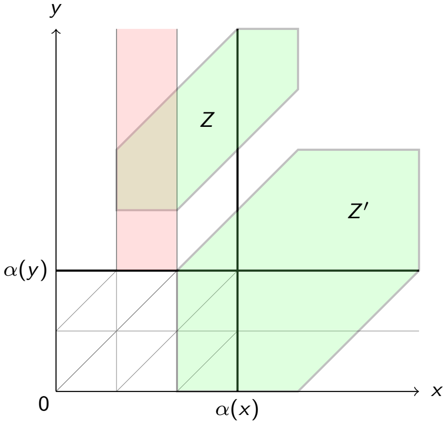
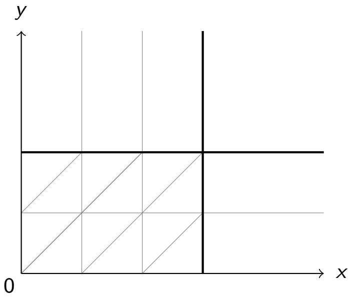


$Z \not\subseteq \text{Closure}_\alpha(Z') \Leftrightarrow \exists R.\ R$ **intersects Z**, R does **not intersect Z′**

Coming next: An **efficient algorithm** for $Z \not\subseteq \text{Closure}_\alpha(Z')$

Step 1: Representing regions and zones



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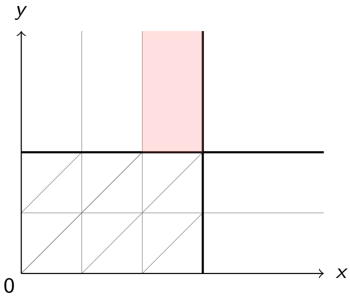

$x < 3$ $\qquad$ $y < \infty$

$x > 2$ $\qquad$ $y > 2$

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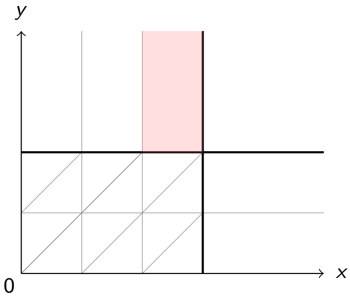

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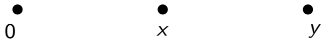

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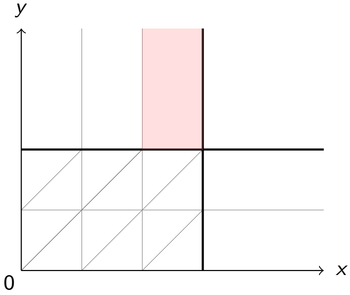

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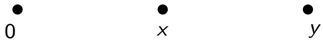

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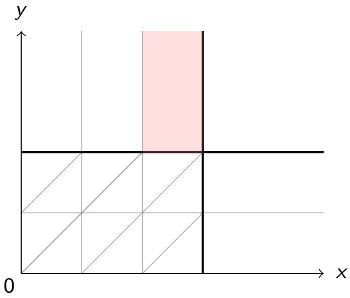

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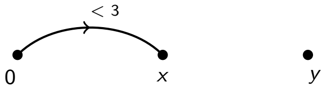

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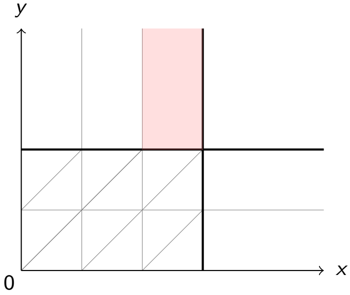

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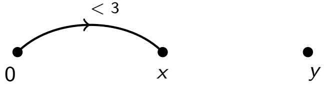

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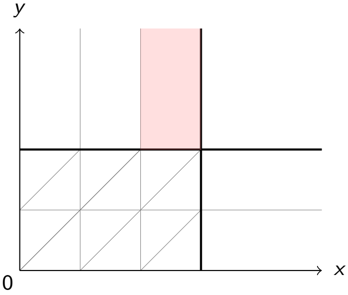
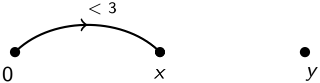


$x - 0 < 3$ $\quad$ $y < \infty$

$0 - x < -2$ $\quad$ $y > 2$



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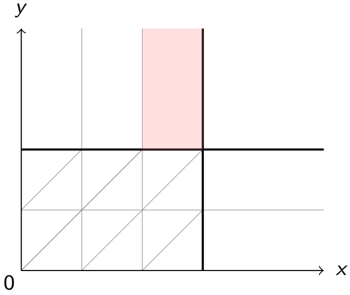

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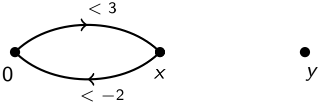

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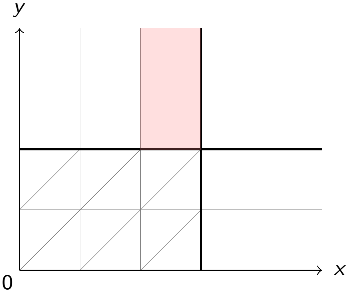

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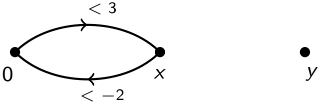

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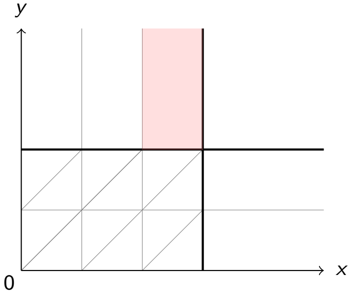

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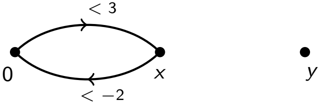

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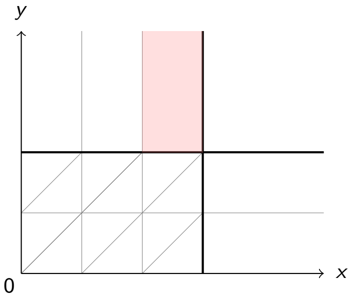

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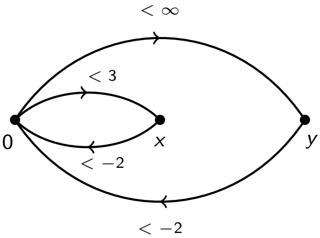

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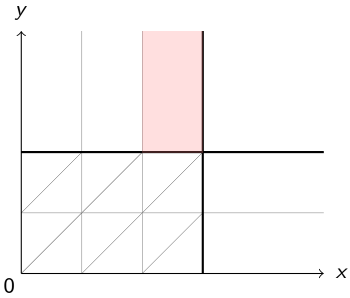

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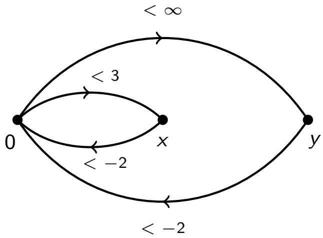

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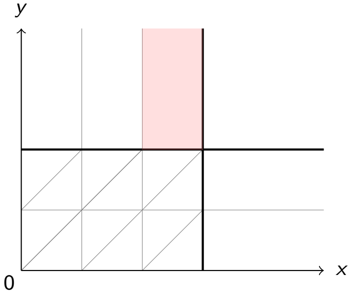

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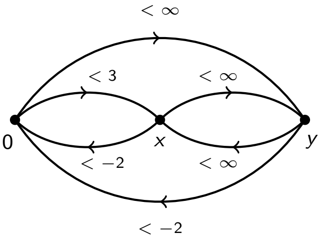

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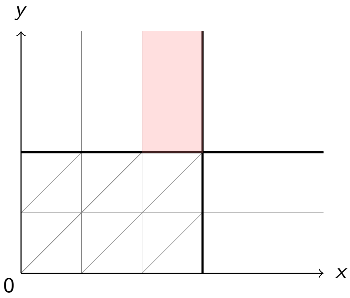

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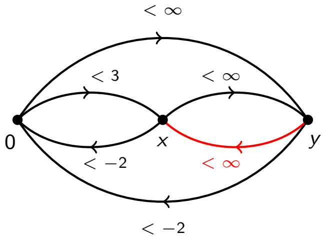
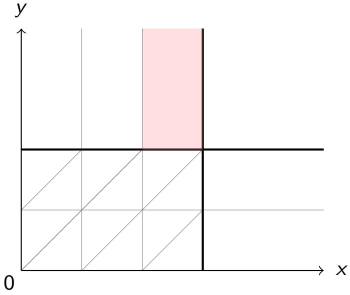


Need a **canonical** representation

Step 1: Representing regions and zones



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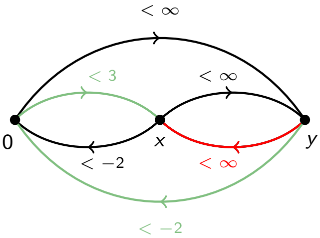
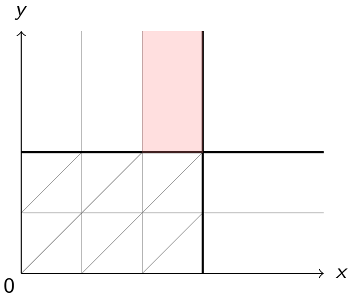


Shortest path should be given by the **direct edge**

Step 1: Representing regions and zones



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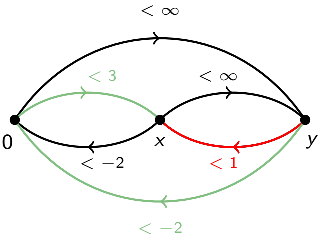

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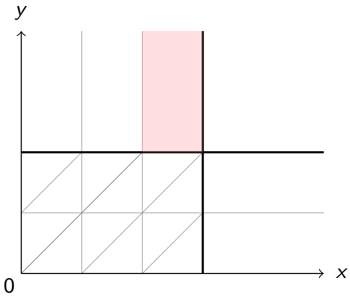

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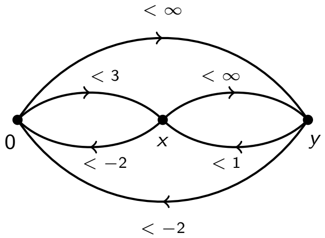

For every zone Z, **canonical distance graph** G_Z

Step 2: When is $R \cap Z'$ empty?

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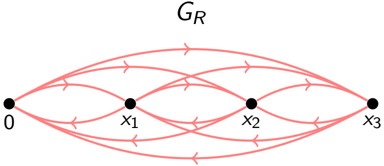
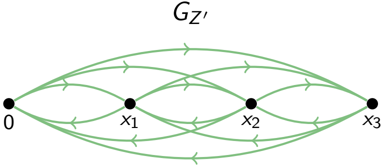
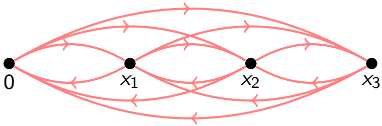


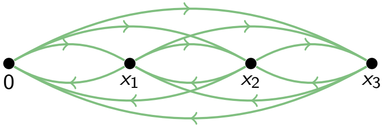


Step 2: When is $R \cap Z'$ empty?

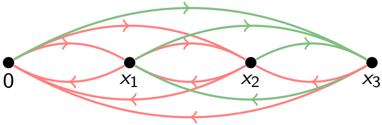
G_R



$G_{Z'}$



$\min(G_R, G_{Z'})$



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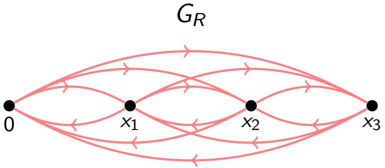
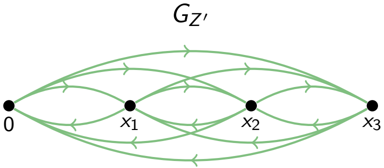
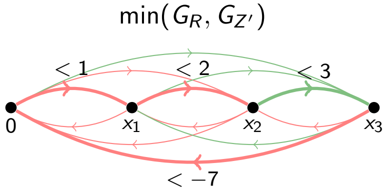






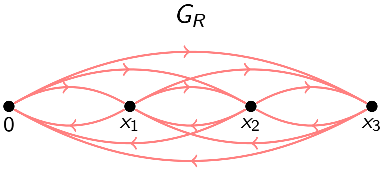
$$\begin{array}{cccc} x_1 & - & 0 & < 1 \\ x_2 & - & x_1 & < 2 \\ x_3 & - & x_2 & < 3 \\ 0 & - & x_3 & < -7 \end{array}$$

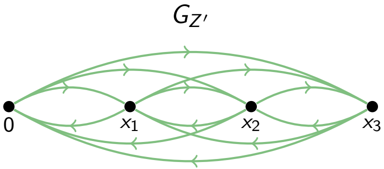
$0 < -1!$

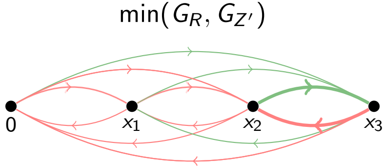
Lemma

$R \cap Z'$ is **empty** $\Leftrightarrow$ $\min(G_R, G_{Z'})$ has a **negative cycle**

Step 2: When is $R \cap Z'$ empty?



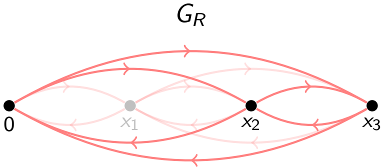


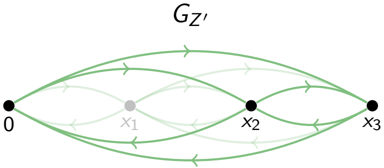


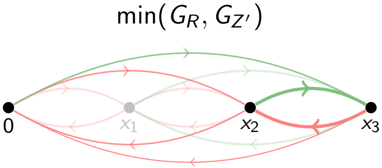
Lemma [Bou04]

$R \cap Z'$ is **empty** $\Leftrightarrow$ $\min(G_R, G_{Z'})$ has a **negative cycle** involving **2 clocks!**

Step 2: When is $R \cap Z'$ empty?







Lemma

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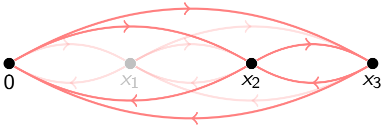
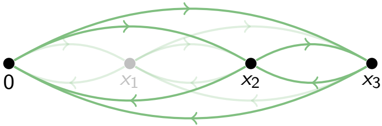
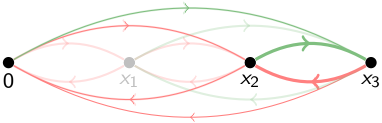
Step 2: When is $R \cap Z'$ empty?

$G_{\mathrm{Proj}_{x_2 x_3}(R)}$



$G_{\mathrm{Proj}_{x_2 x_3}(Z')}$





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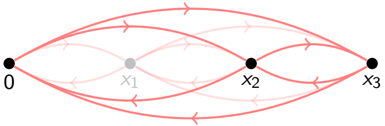
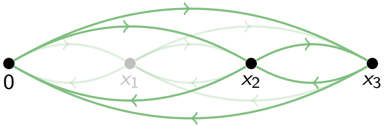
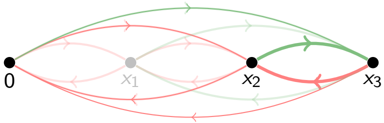


$G_{\mathrm{Proj}_{x_2 x_3}(Z')}$



$\min\left(G_{\mathrm{Proj}_{x_2 x_3}(R)}, G_{\mathrm{Proj}_{x_2 x_3}(Z')}\right)$



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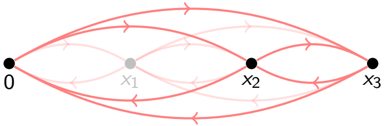
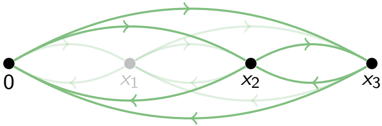


$G_{\mathrm{Proj}_{x_2 x_3}(Z')}$



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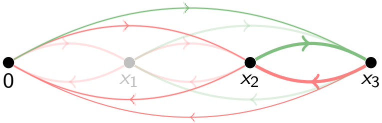

Lemma

$R \cap Z'$ is **empty** $\Leftrightarrow$ $\exists\, x, y.\ \mathrm{Proj}_{\mathbf{xy}}(R) \cap \mathrm{Proj}_{\mathbf{xy}}(Z')$ is **empty**

Step 3: Reduction to two clocks

Recall: $Z \not\subseteq \mathrm{Closure}_\alpha(Z') \Leftrightarrow \exists R.\ R$ intersects Z, R does not intersect Z'

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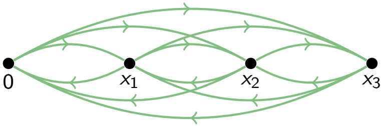
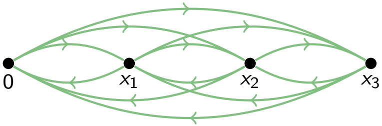


Z



Z'

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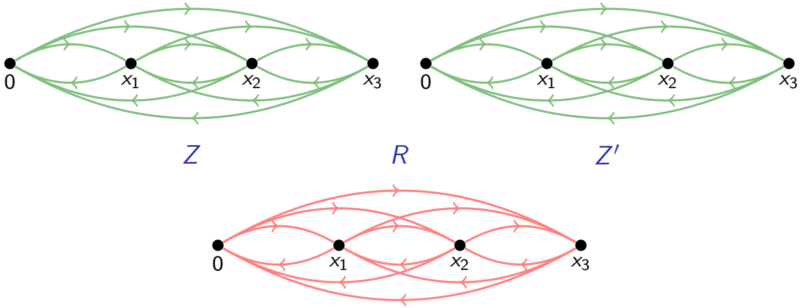

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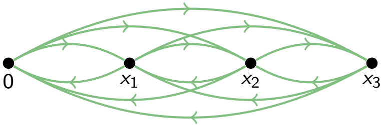
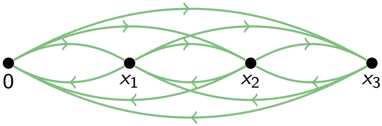
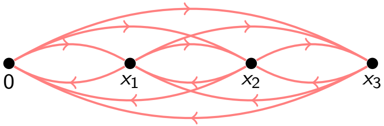




$$Z \quad \cap \quad R \quad \not\cap \quad Z'$$



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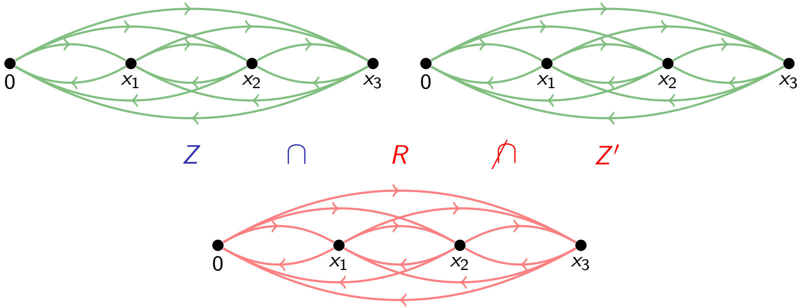

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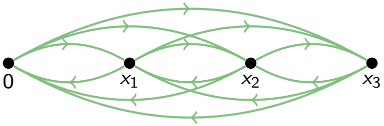
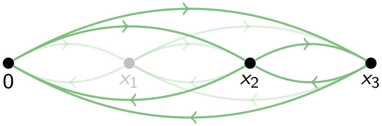
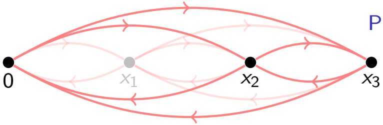




$Z \qquad \cap \qquad R \qquad \not\cap \qquad Z'$



$\text{Proj}_{x_2x_3}(R) \not\cap \text{Proj}_{x_2x_3}(Z')$

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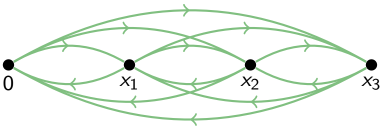

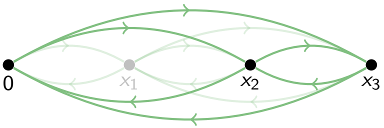

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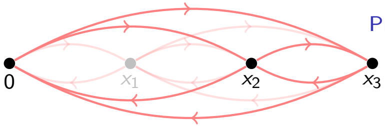

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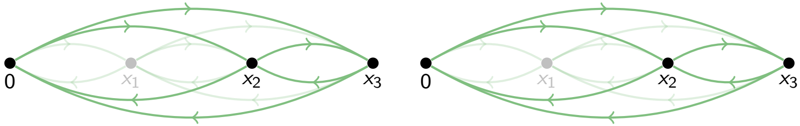

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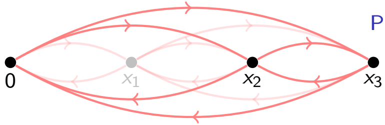
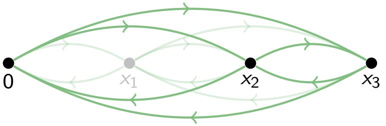
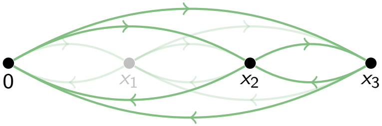


$\text{Proj}_{x_2 x_3}(R) \not\cap \text{Proj}_{x_2 x_3}(Z')$

Step 3: Reduction to two clocks

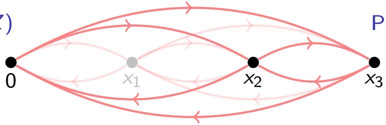
Recall: $Z \not\subseteq \text{Closure}_\alpha(Z') \Leftrightarrow \exists R.\ R$ intersects Z, R does not intersect Z'





$Z \quad \cap \quad R \quad \not\cap \quad Z'$

$\text{Proj}_{x_2x_3}(R) \cap \text{Proj}_{x_2x_3}(Z)$



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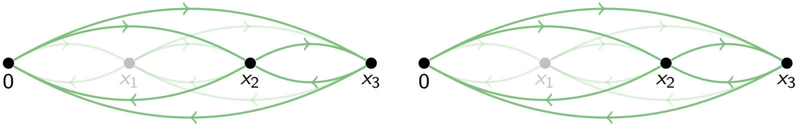

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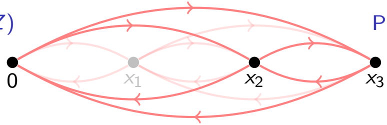


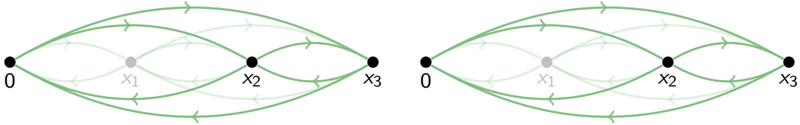
$\text{Proj}_{x_2x_3}(R) \not\cap \text{Proj}_{x_2x_3}(Z')$

Theorem

$Z \not\subseteq \text{Closure}_\alpha(Z')$ if and only if there **exist 2 clocks** x, y s.t.

$$\mathbf{Proj_{xy}}(Z) \not\subseteq \text{Closure}_\alpha(\mathbf{Proj}_{xy}(Z'))$$

Step 3: Reduction to two clocks



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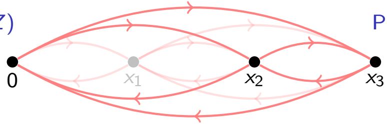

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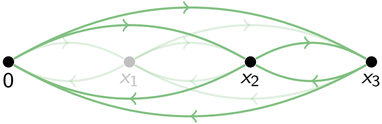
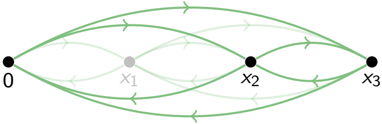
Slightly **modified edge-edge comparison** is enough

Step 3: Reduction to two clocks





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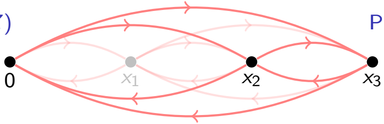

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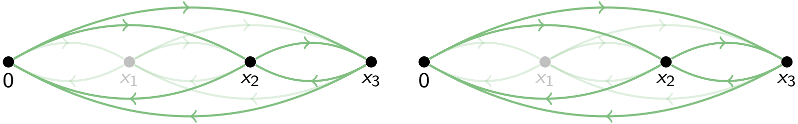
Complexity: $\mathcal{O}(|X|^2)$, where X is the set of clocks

Step 3: Reduction to two clocks



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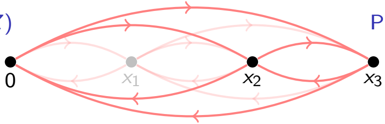

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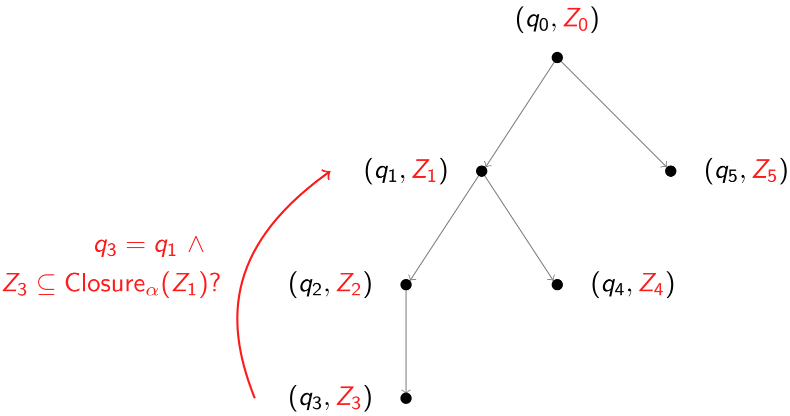
Theorem

$Z \not\subseteq \mathrm{Closure}_\alpha(Z')$ if and only if there **exist 2 clocks** x, y s.t.

$$\mathbf{Proj_{xy}}(Z) \not\subseteq \mathrm{Closure}_\alpha(\mathbf{Proj}_{xy}(Z'))$$

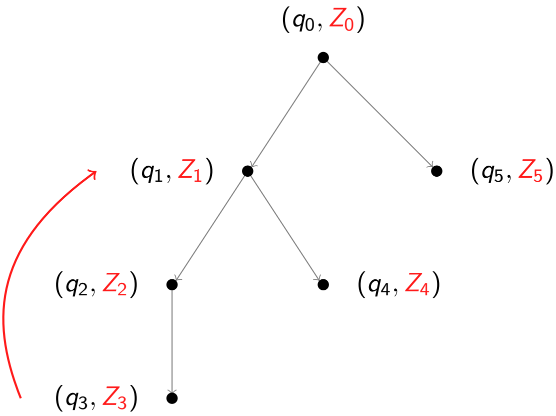
Same complexity as $Z \subseteq Z'$!

So what do we have now...



Efficient algorithm for $Z \subseteq \mathrm{Closure}_\alpha(Z')$

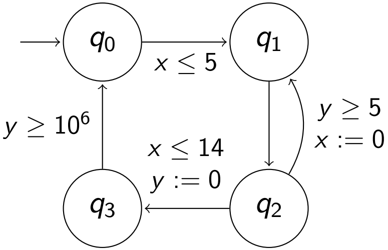
So what do we have now...



$q_3 = q_1 \wedge$
$Z_3 \subseteq \text{Closure}_\alpha(Z_1)$?

Coming next: **prune** the **bound function** α!

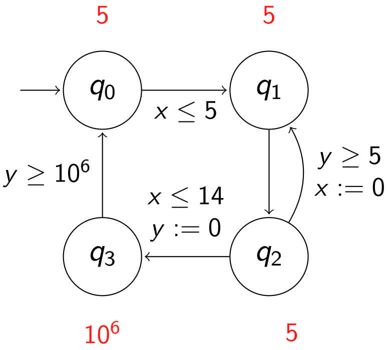
Bound function α



Naive: $\alpha(x) = 14$, $\alpha(y) = 10^6$

Size of graph $\sim 10^5$

Static analysis: bound function for every q [BBFL03]



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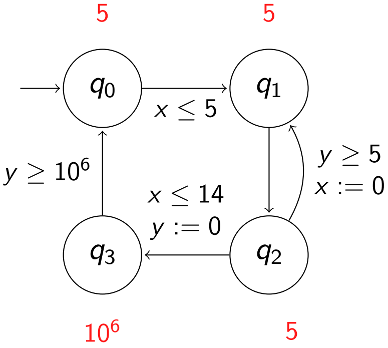
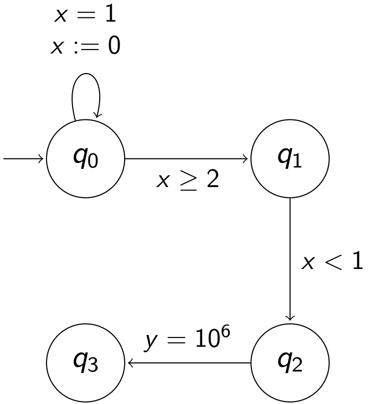


Naive: $\alpha(x) = 14$, $\alpha(y) = 10^6$

But this is not enough!

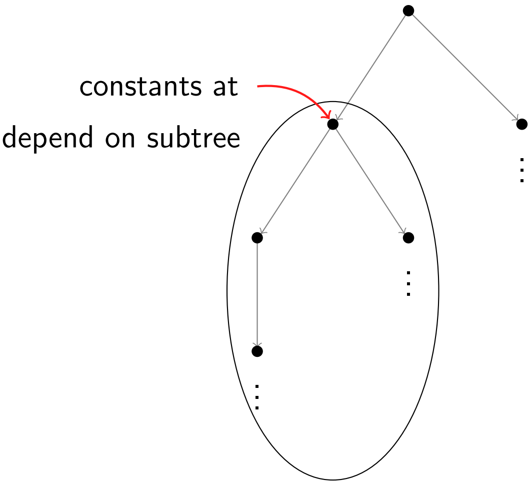
Need to look at semantics...

Static analysis: $\alpha(y) = 10^6$



More than $\mathbf{10^6}$ zones at q_0 **not necessary**!

Bound function for every (q, Z) in $ZG(\mathcal{A})$



Constant propagation

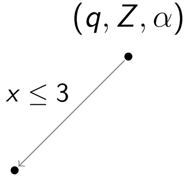
$\alpha(x) = -\infty$

$$(q, Z, \alpha)$$

•

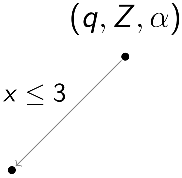
Constant propagation

$\alpha(x) = -\infty$



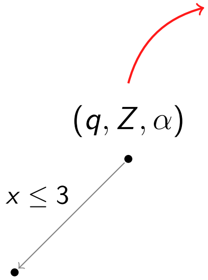
Constant propagation

$\alpha(x) = \ 3$



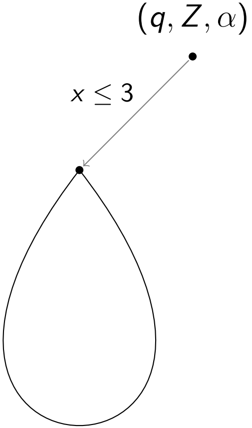
Constant propagation

$\alpha(x) = \ 3$



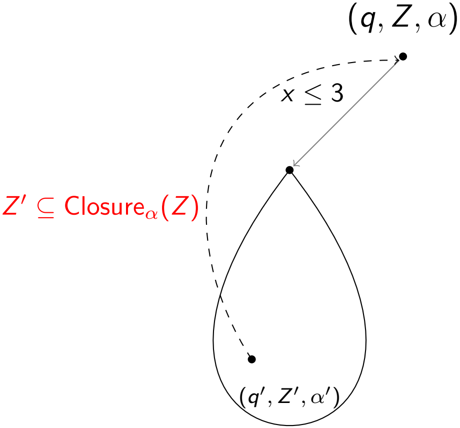
Constant propagation

$\alpha(x) = \quad 5$



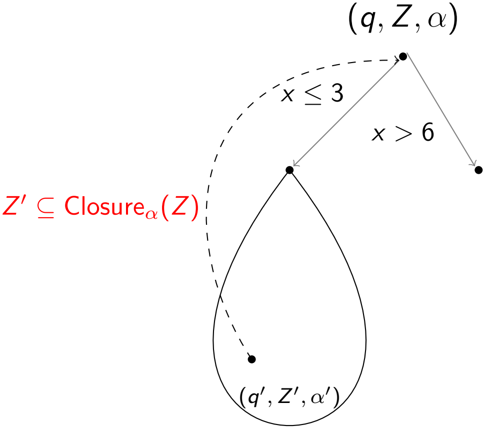
Constant propagation

$\alpha(x) = \quad 5$

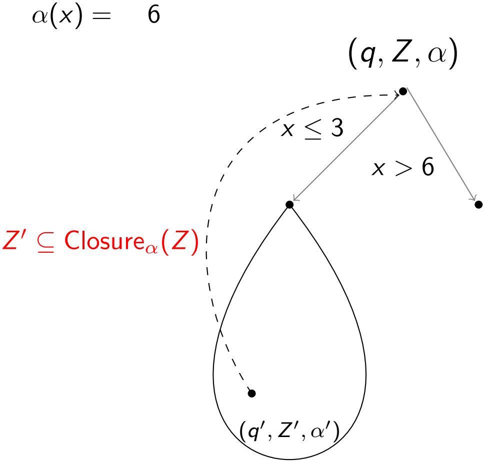


Constant propagation

$\alpha(x) = \quad 5$

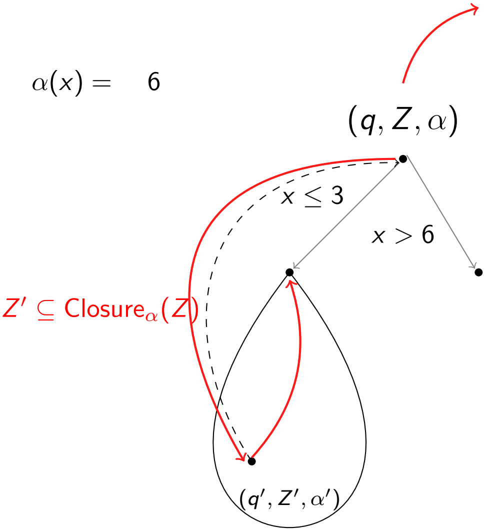


Constant propagation



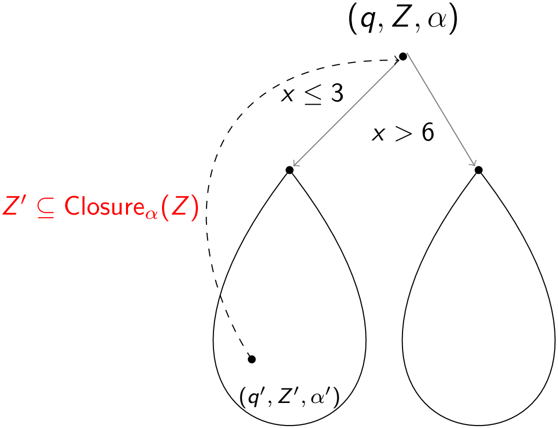
Constant propagation

$\alpha(x) = \quad 6$



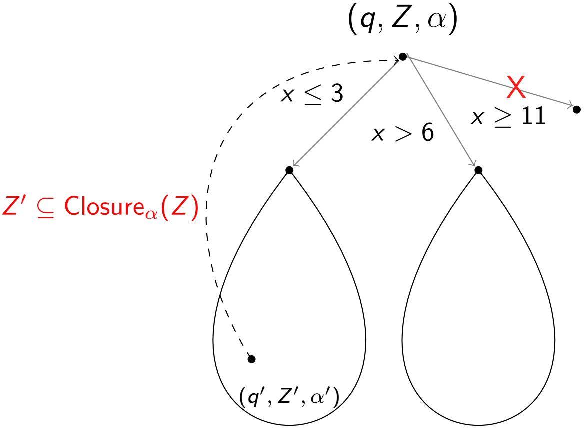
Constant propagation

$\alpha(x) = \quad 6$



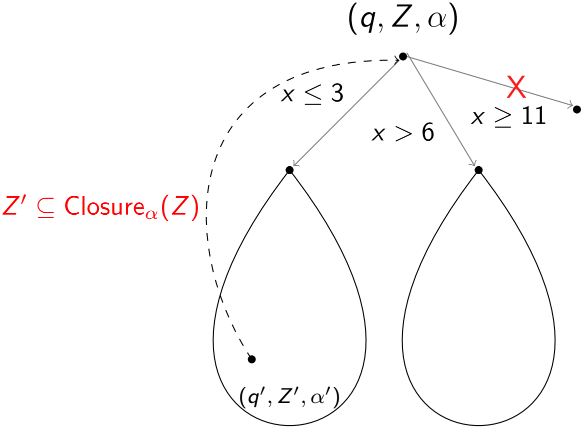
Constant propagation

$\alpha(x) = \quad 6$



Constant propagation

$\alpha(x) = \quad 11$



Constant propagation

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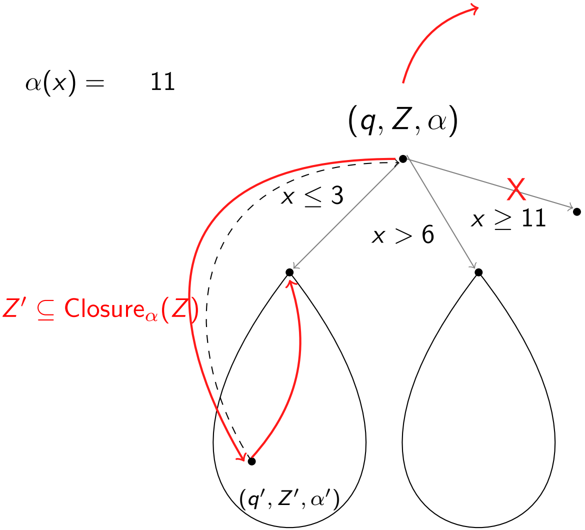

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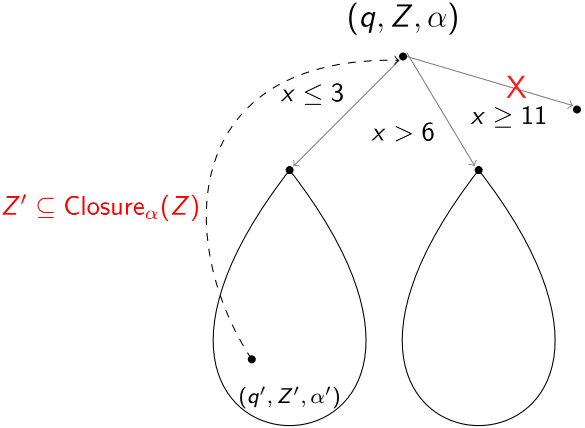
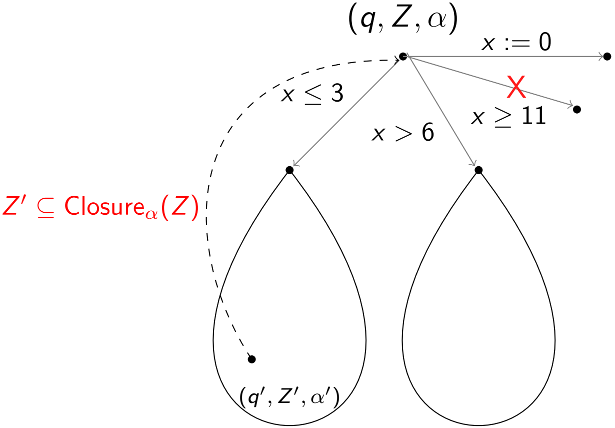


Constant propagation

$\alpha(x) = \quad 11$



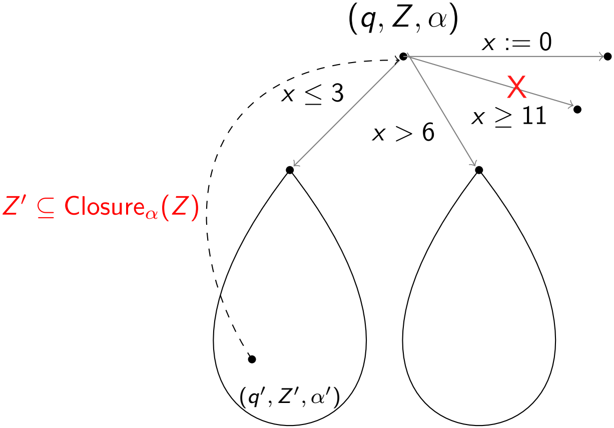
Constant propagation

All **tentative nodes** consistent

+ No more **exploration**

→ Terminate!

$\alpha(x) = \quad 11$



Invariants on the bounds

- Non tentative nodes: $\alpha = max\{\alpha_{succ}\}$ (modulo resets)
- Tentative nodes: $\alpha = \alpha_{covering}$

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Theorem (Correctness)

An accepting state is reachable in $ZG(\mathcal{A})$ iff the algorithm reaches a node with an accepting state and a non-empty zone.

Overall algorithm

- Compute $ZG(\mathcal{A})$: $Z \subseteq \mathrm{Closure}_{\alpha'}(Z')$ for **termination**
- **Bounds** α calculated **on-the-fly**
- Abstraction Extra^{+}_{LU} can **also** be **handled**:

$$
\begin{tikzcd}
 & \mathfrak{a}_{\preccurlyeq LU} & \\
 & \mathrm{Closure}_{\alpha} \circ \mathrm{Extra}^{+}_{LU} \arrow[u] & \\
\mathrm{Closure}_{\alpha} \arrow[ur] & & \mathrm{Extra}^{+}_{LU} \arrow[ul] \\
 & \mathrm{Extra}^{+}_{\alpha} \arrow[ul] \arrow[ur] & \\
 & \mathrm{Extra}_{\alpha} \arrow[u] &
\end{tikzcd}
$$

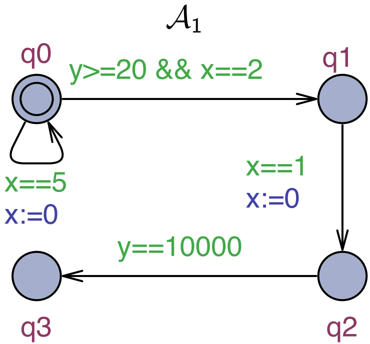
An **efficient** $\mathcal{O}(|X|^2)$ procedure for $Z \subseteq \mathrm{Closure}_{\alpha}(\mathit{Extra}^{+}_{LU}(Z'))$!

Benchmarks

Model	Our algorithm		UPPAAL's algorithm		UPPAAL 4.1.3 (-n4 -C -o1)	
	nodes	s.	nodes	s.	nodes	s.
CSMA/CD7	5031	0.32	5923	0.27	–	T.O.
CSMA/CD8	16588	1.36	19017	1.08	–	T.O.
CSMA/CD9	54439	6.01	60783	4.19	–	T.O.
FDDI10	459	0.02	525	0.06	12049	2.43
FDDI20	1719	0.29	2045	0.78	–	T.O.
FDDI30	3779	1.29	4565	4.50	–	T.O.
Fischer7	7737	0.42	18374	0.53	18374	0.35
Fischer8	25080	1.55	85438	2.48	85438	1.53
Fischer9	81035	5.90	398685	12.54	398685	8.95
Fischer10	–	T.O.	–	T.O.	1827009	53.44

- **Extra**$^+_{LU}$ and **static** analysis bounds in UPPAAL
- **Closure**$_\alpha$**(Extra**$^+_{LU}$**)** and **otf** bounds in our algorithm

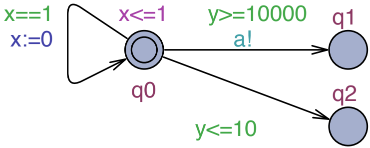
Experiments I



$\mathcal{A}_1$	nodes	s.
Our algorithm	7	0.0
UPPAAL's algorithm	2003	0.60
UPPAAL 4.1.3	2003	0.01

Experiments II

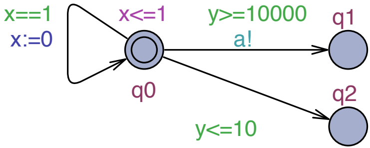
$\mathcal{A}_2$



$\mathcal{A}_2$	nodes	s.
Our algorithm	2	0.0
UPPAAL's algorithm	10003	0.07
UPPAAL 4.1.3	10003	0.07

Experiments II

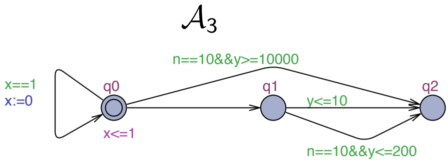
$\mathcal{A}_2$



$\mathcal{A}_2$	nodes	s.
Our algorithm	2	0.0
UPPAAL's algorithm	10003	0.07
UPPAAL 4.1.3	10003	0.07

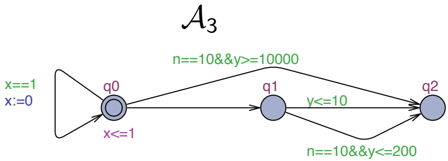
Occurs in **CSMA/CD**!

Experiments III



$\mathcal{A}_3$	nodes	s.
Our algorithm	3	0.0
UPPAAL's algorithm	10004	0.37
UPPAAL 4.1.3	10004	0.32

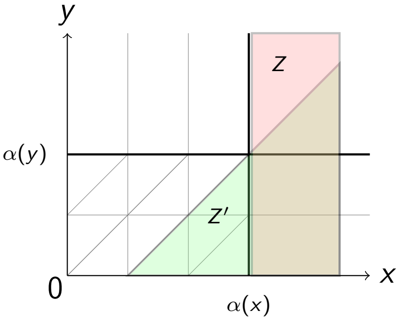
Experiments III



$\mathcal{A}_3$	nodes	s.
Our algorithm	3	0.0
UPPAAL's algorithm	10004	0.37
UPPAAL 4.1.3	10004	0.32

Occurs in **Fischer**!

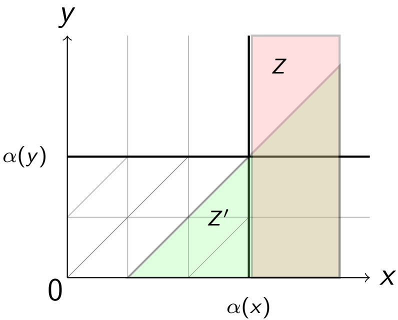
Experiments IV



Z' : $x - y \geq 1$

Z : $x > \alpha(x)$

Experiments IV



$Z' : \; x - y \geq 1$
$Z : \; x > \alpha(x)$

Occurs in **FDDI**!

Conclusions & Perspectives

- **Efficient implementation** of a non-convex approximation that **subsumes** current ones in use
- **On-the-fly learning** of bounds that is **better** than the current static analysis

- More **sophisticated** non-convex approximations
- Propagating **more** than constants
- Automata with **diagonal** constraints

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