Topics in Timed Automata

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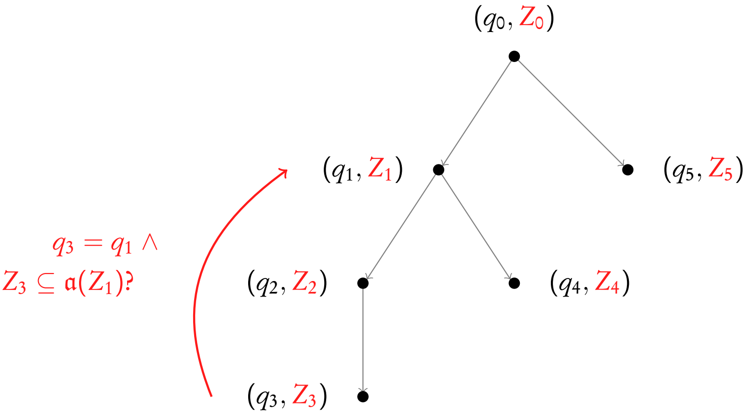
Software modeling and Verification group

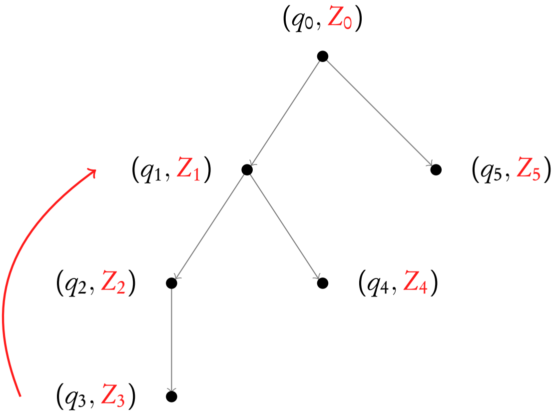
Reachability for timed automata

Key idea: Compute the **zone graph**, use **abstraction** for termination



Reachability for timed automata

Key idea: Compute the **zone graph**, use **abstraction** for termination

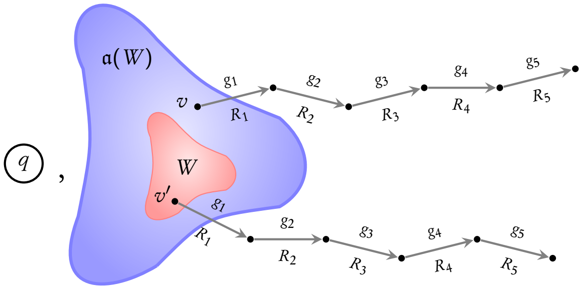


$q_3 = q_1 \wedge$
$Z_3 \subseteq \mathfrak{a}(Z_1)$?

Coarser the abstraction, **smaller** the zone graph

Condition 1: $\mathfrak{a}$ should have **finite range**

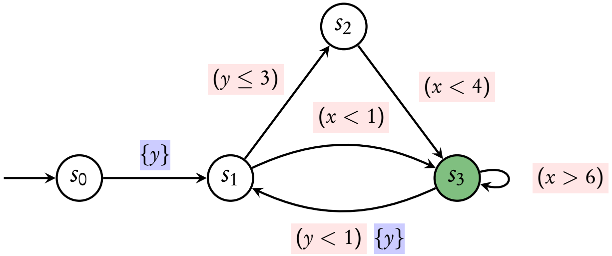
Condition 2: $\mathfrak{a}$ should be sound $\Rightarrow \mathfrak{a}(W)$ can contain only valuations **simulated** by W



Bounds and abstractions

Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**



Bounds and abstractions

Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**

$(y \leq 3)$

$(x < 4)$

$(x < 1)$

$(x > 6)$

$(y < 1)$

Bounds and abstractions

Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**

$(y \leq 3)$

$(x < 4)$

$(x < 1)$

$(x > 6)$

$(y < 1)$

M-bounds [AD94]

$$M(x) = 6,\ M(y) = 3$$

$$v \preccurlyeq_M v'$$

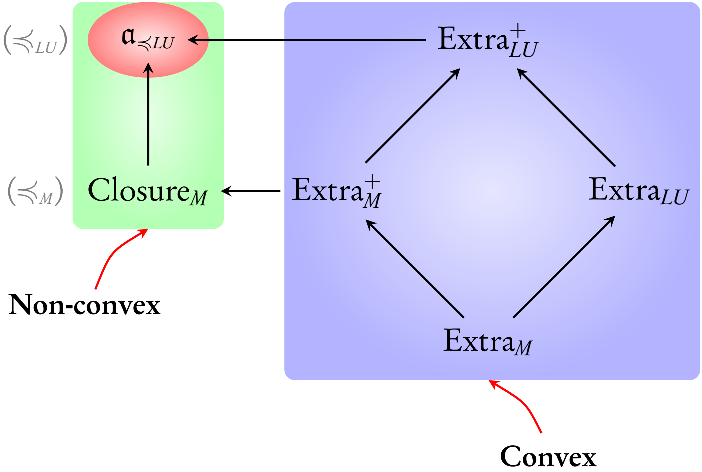
LU-bounds [BBLP04]

$$L(x) = 6,\ L(y) = -\infty$$

$$U(x) = 4,\ U(y) = 3$$

$$v \preccurlyeq_{LU} v'$$

Abstractions in literature [BBLP04, Bou04]



Last lecture: **Efficiently** using the M-bounds based Closure_M abstraction

Lecture 7:

Lower-upper bounds for abstraction

LU-guards: guards consistent with given L and U

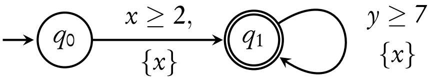
LU-guards for $L(x) = 3, U(x) = 5, L(y) = 8, U(y) = -\infty$

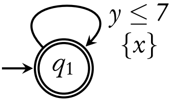
$$x \geq 0, x \geq 1, x \geq 2, x \geq 3$$
$$x \leq 0, x \leq 1, \ldots, x \leq 5$$
$$y \geq 0, y \geq 1, \ldots, y \geq 8$$

(same with $<$ and $>$)

LU-automata: automata with only LU-guards

$$L(x) = 3, U(x) = 5, L(y) = 8, U(y) = -\infty$$





LU-automata: automata with only LU-guards

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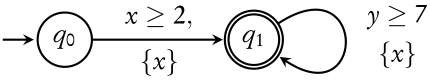
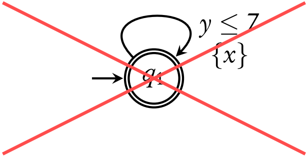
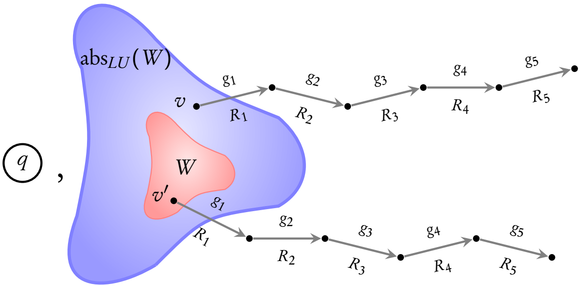




What do we need?

1. An abstraction abs_{LU} that is **sound** and **complete** for **all** LU-automata



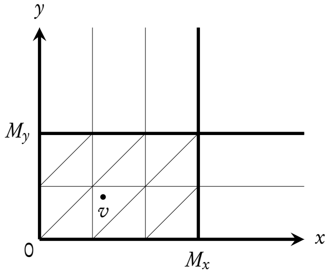
2. **Efficient** inclusion testing $Z \subseteq abs_{LU}(Z')$

Step 1:

LU-regions

Classic regions [AD94] : v' belongs to $[v]^M$ if:

- **Invariance by guards:** v' satisfies the same guards as v,
- **Invariance by time-elapse:** for every time elapse $\delta \in \mathbb{R}_{\geq 0}$, there is a $\delta' \in \mathbb{R}_{\geq 0}$ such that $v' + \delta' \in [v + \delta]^M$.



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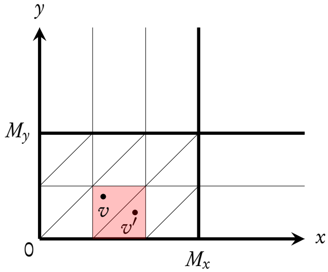

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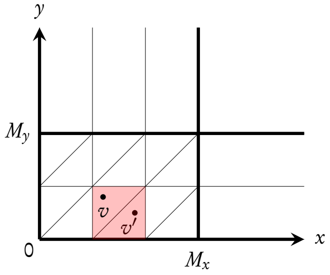

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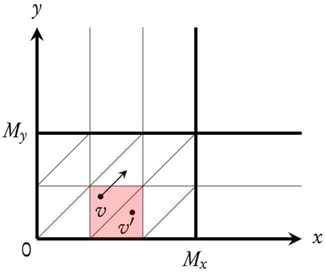

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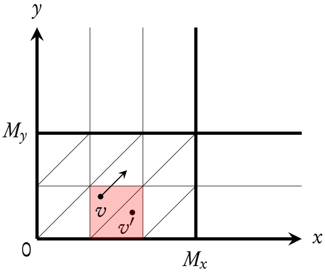

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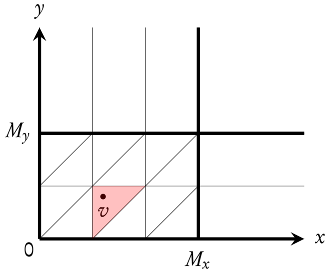

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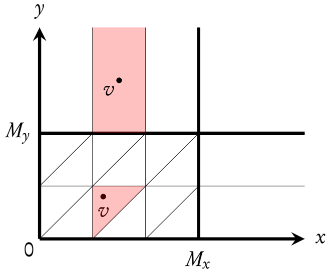

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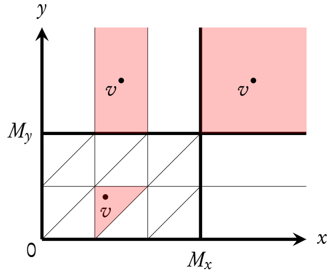


Classic regions [AD94] **:** Given M, v' belongs to $[v]^M$ if:

- **Invariance by guards:** v' satisfies the same guards as v,
- **Invariance by time-elapse:** for every pair of clocks x, y with:

$$v(x) \leq M_x, \quad v(y) \leq M_y$$
$$\lfloor v(x) \rfloor = \lfloor v'(x) \rfloor \text{ and } \lfloor v(y) \rfloor = \lfloor v'(y) \rfloor$$

  we have:

  - if $0 < \{v(x)\} < \{v(y)\}$, then $0 < \{v'(x)\} < \{v'(y)\}$
  - if $0 < \{v(x)\} = \{v(y)\}$, then $0 < \{v'(x)\} = \{v'(y)\}$

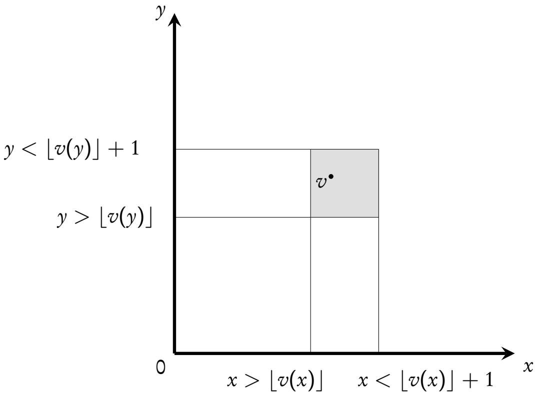
$\lfloor v(x) \rfloor$: integer part of $v(x)$

$\{v(x)\}$: fractional part of $v(x)$

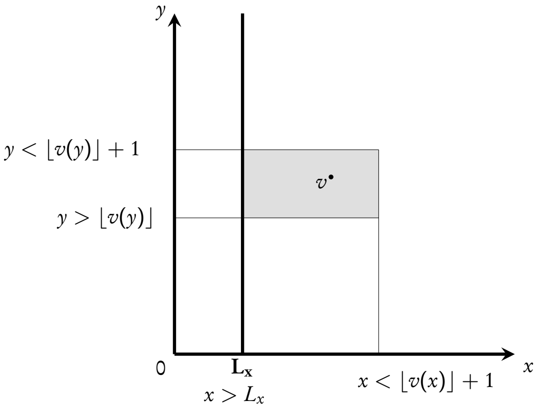
Coming next...

Regions for the **LU**-case

Invariance by (LU-) guards: $v(x)$ is less than both L_x, U_x



Invariance by (LU-) guards: $v(x) > L_x$



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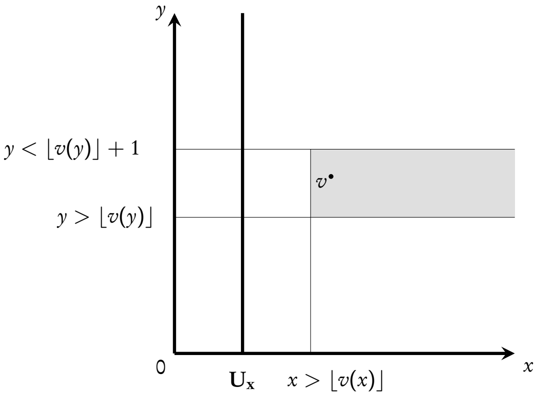
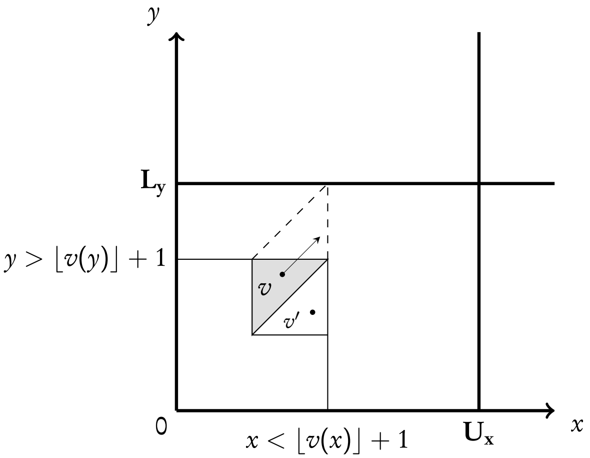
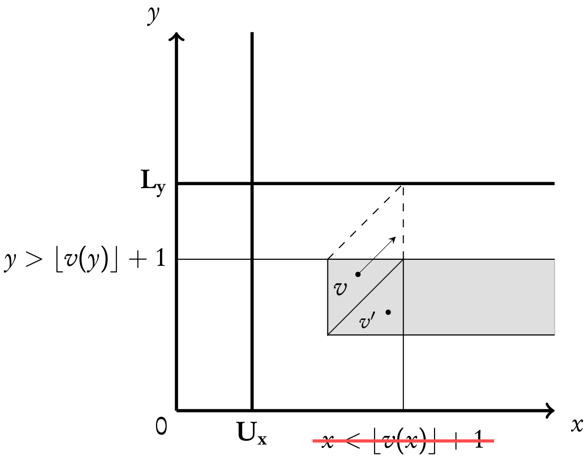


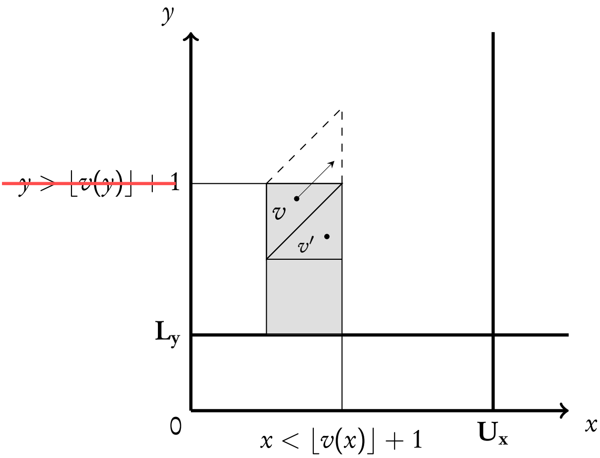
Invariance by time-elapse: $v(x) \leq U_x$, $v(y) \leq L_y$



Invariance by time-elapse: $v(x) > U_x, \quad v(y) \le L_y$



Invariance by time-elapse: $v(x) \leq U_x, \quad v(y) > L_y$



LU-regions

Definition: v' belongs to $\langle v \rangle^{LU}$ if:

- **Invariance by guards:** v' satisfies the same guards as v,
- **Invariance by time-elapse:** for every pair of clocks x, y with:

$$v(x) \leq U_x,\ v(y) \leq L_y$$
$$\lfloor v(x) \rfloor = \lfloor v'(x) \rfloor \text{ and } \lfloor v(y) \rfloor = \lfloor v'(y) \rfloor,$$

  we have:

  - if $\mathtt{0} < \{v(x)\} < \{v(y)\}$, then $\mathtt{0} < \{v'(x)\} < \{v'(y)\}$
  - if $\mathtt{0} < \{v(x)\} = \{v(y)\}$, then $\mathtt{0} < \{v'(x)\} = \{v'(y)\}$

Step 2:

An abstraction abs_{LU}

$$v \sqsubseteq_{LU} v'$$

if

$$\exists \delta' \in \mathbb{R}_{\geq 0} \text{ s.t. } v' + \delta' \in \langle v \rangle^{LU}$$

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Definition

$$\mathrm{abs}_{LU}(W) = \{v \mid \exists v' \in W \text{ s.t. } v \sqsubseteq_{LU} v'\}$$

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Definition

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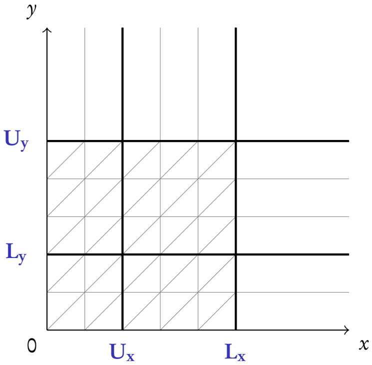
abs_{LU} is **sound** and **complete**

Example



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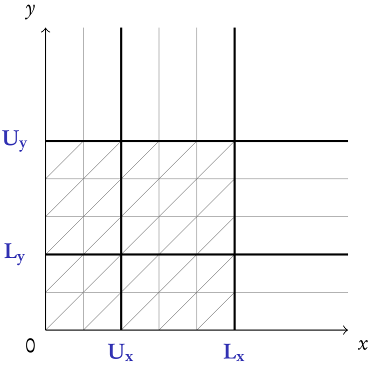

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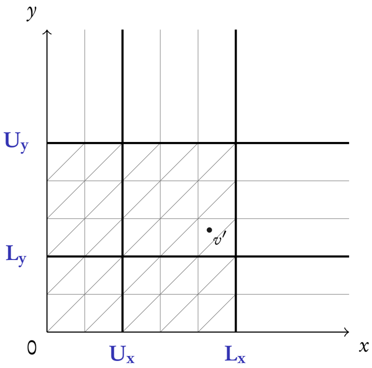

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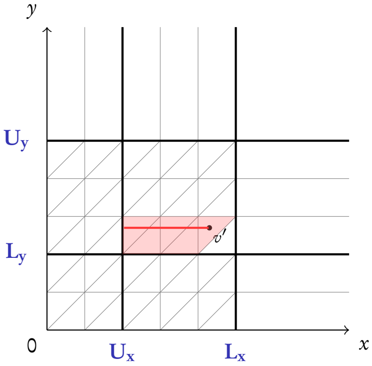

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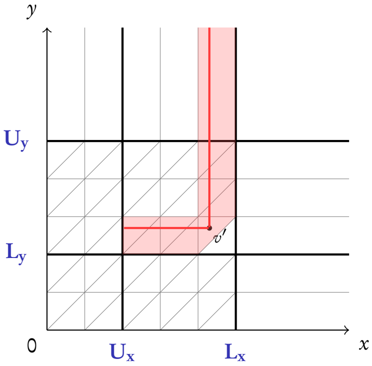

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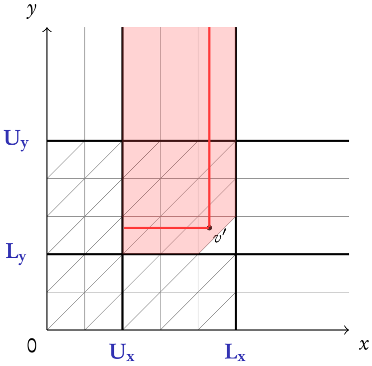

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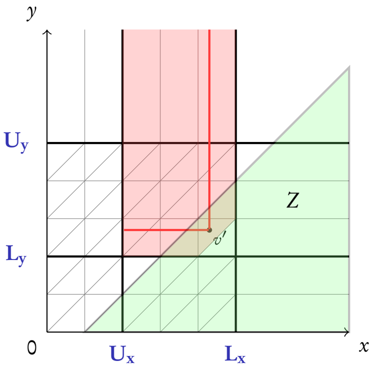

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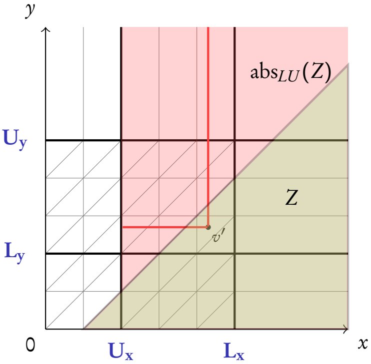

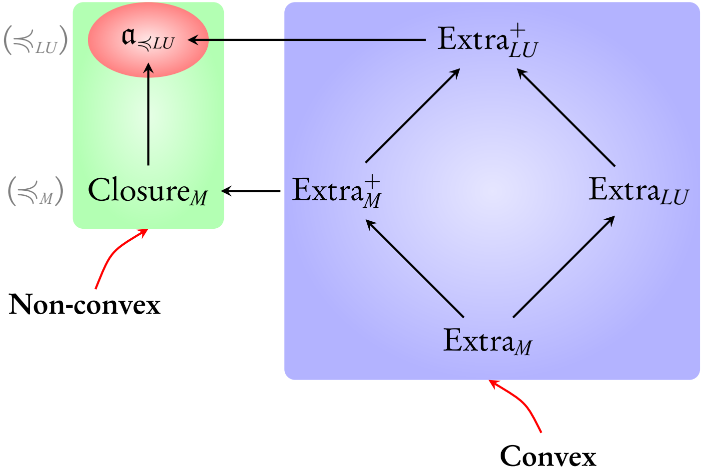

$(\preccurlyeq_{LU})$
$\mathfrak{a}_{\preccurlyeq LU}$
Extra^{+}_{LU}
$(\preccurlyeq_{M})$
$\mathrm{Closure}_M$
Extra^{+}_{M}
Extra_{LU}
Non-convex
Extra_M
Convex

Time-elapsed zone Z: if $v \in Z$, then $v + \delta \in Z$ for all $\delta \in \mathbb{R}_{\geq 0}$

$\mathfrak{a}_{\preccurlyeq LU}$ coincides with abs_{LU}

If Z is time-elapsed, then $\mathfrak{a}_{\preccurlyeq LU}(Z) = \mathrm{abs}_{LU}(Z)$

Better abstractions for timed automata

F. Herbreteau, B. Srivathsan, I. Walukiewicz. *LICS'12*

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If Z is time-elapsed, then $\mathfrak{a}_{\preccurlyeq LU}(Z) = \mathrm{abs}_{LU}(Z)$

Optimality

$\mathfrak{a}_{\preccurlyeq LU}(Z)$ is the **coarsest** abstraction that is **sound** and **complete** for all LU-automata

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Step 3:

Efficient inclusion

$$v \sqsubseteq_{LU} v'$$

if

$$\exists \delta' \in \mathbb{R}_{\geq 0} \text{ s.t. } v' + \delta' \in \langle v \rangle^{LU}$$

Definition

$$\mathrm{abs}_{LU}(W) = \{v \mid \exists v' \in W \text{ s.t. } v \sqsubseteq_{LU} v'\}$$

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Definition

$$\mathrm{abs}_{LU}(W) = \{v \mid \exists v' \in W \text{ s.t. } v \sqsubseteq_{LU} v'\}$$

Z, Z': time-elapsed zones

$Z \not\subseteq \mathrm{abs}_{LU}(Z')$ iff there **exists** $v \in Z$ s.t.
$\langle v \rangle^{LU}$ **does not intersect** Z'

Efficient inclusion testing

Reduction to two clocks

$Z \not\subseteq \mathfrak{a}_{\preccurlyeq LU}(Z')$ if and only if there **exist 2 clocks** x, y s.t.

$$\mathbf{Proj}_{xy}(Z) \not\subseteq \mathfrak{a}_{\preccurlyeq LU}(\mathbf{Proj}_{xy}(Z'))$$

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Complexity: $\mathcal{O}(|X|^2)$, where X is the set of clocks

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Same complexity as $Z \subseteq Z'$!

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Same complexity as $Z \subseteq Z'$!

Slightly modified comparison works!

Better abstractions for timed automata

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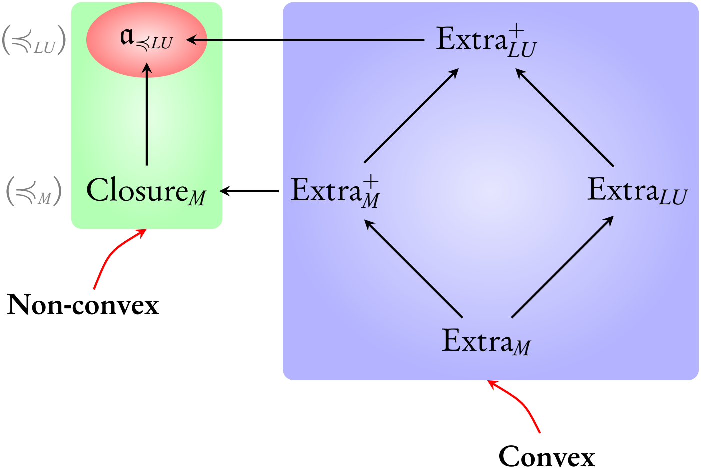

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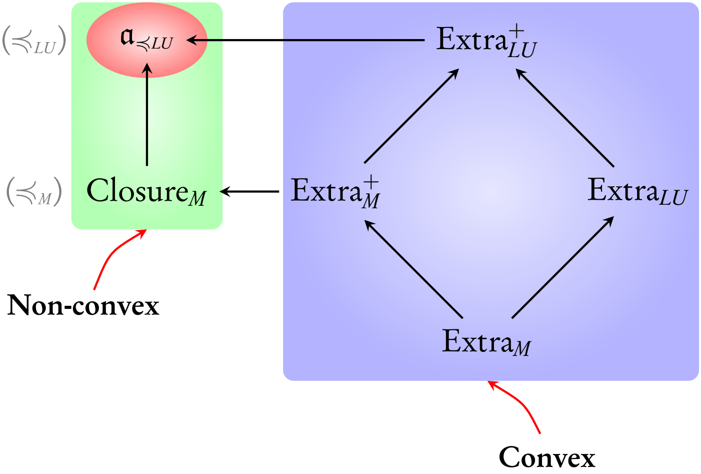
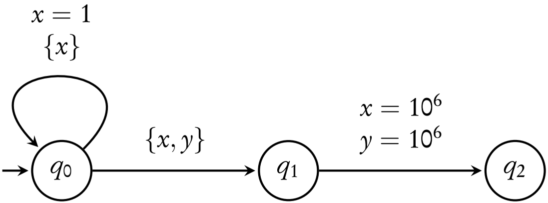


Question: If $\mathfrak{a}_{\preccurlyeq LU}$ is best, can we do better?

Get better **LU-bounds!**

Global LU-bounds

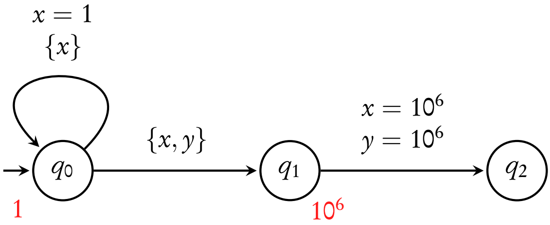


Naive: $L_x = U_x = 10^6$, $L_y = U_y = 10^6$

Size of graph $\sim 10^6$

Static analysis: bounds for every q

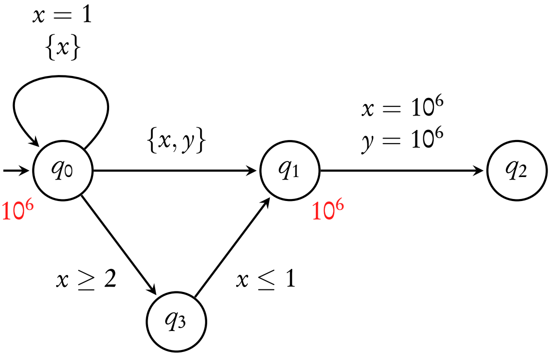
[BBFL03]



Size of graph < 10

Static analysis: bounds for every q

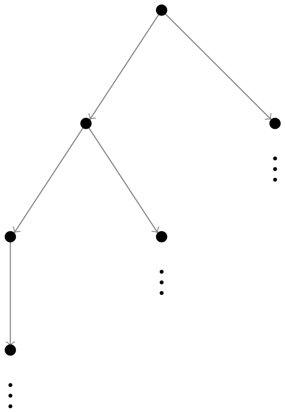
[BBFL03]



Size of graph $\sim 10^6$

Need to look at semantics...

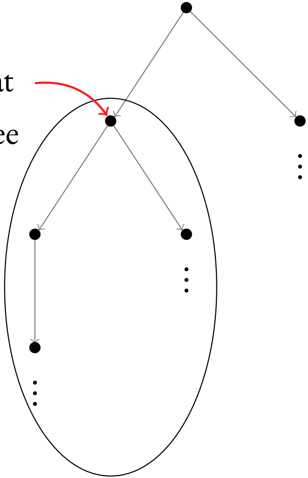
LU bounds for every (q, Z) in zone graph



LU bounds for every (q, Z) in zone graph

constants at

depend on subtree

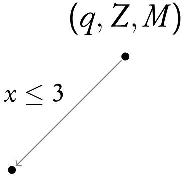


$M(x) = -\infty$

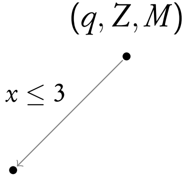
$$(q, Z, M)$$

$\bullet$

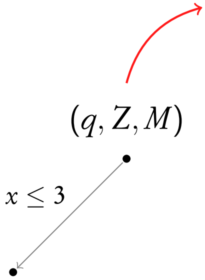
$M(x) = -\infty$



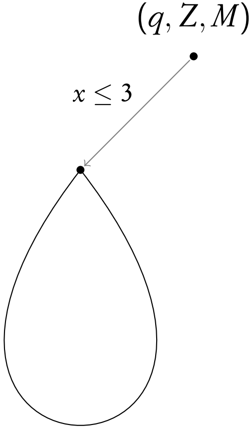
$M(x) = 3$



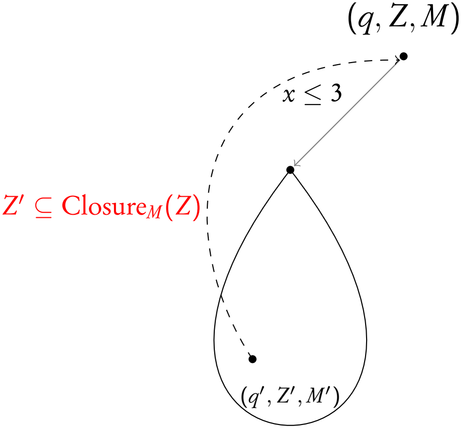
$M(x) = 3$



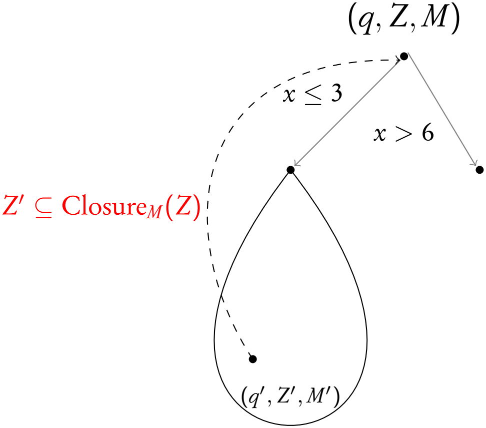
$M(x) = 5$



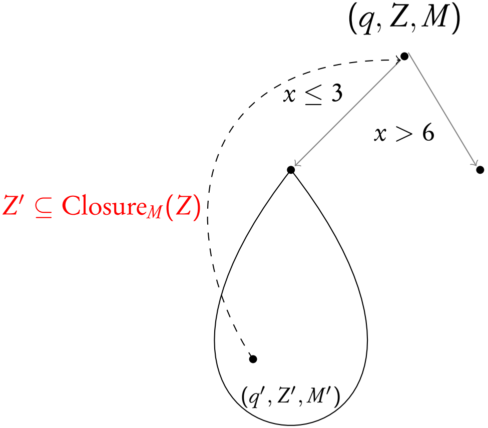
$M(x) = 5$

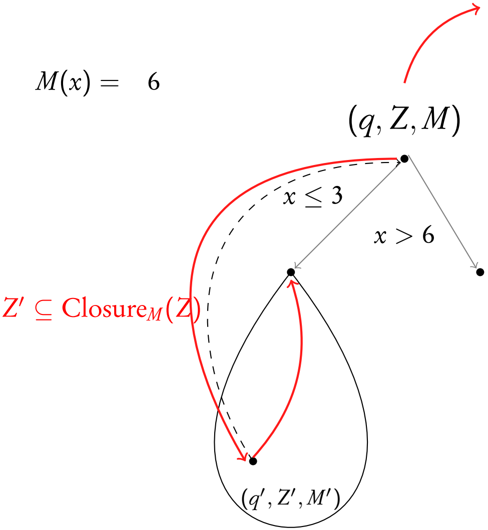


$M(x) = \ 5$

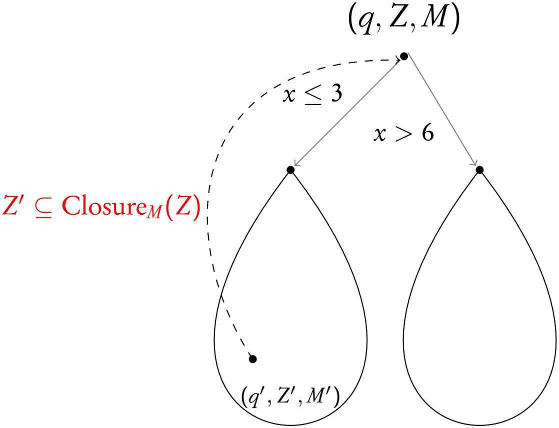


$M(x) = \quad 6$

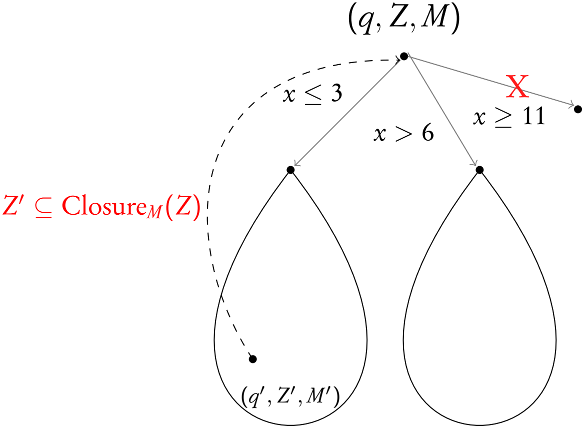



$M(x) = \quad 6$
(q, Z, M)
$x \leq 3$
$x > 6$
$Z' \subseteq \text{Closure}_M(Z)$
(q', Z', M')

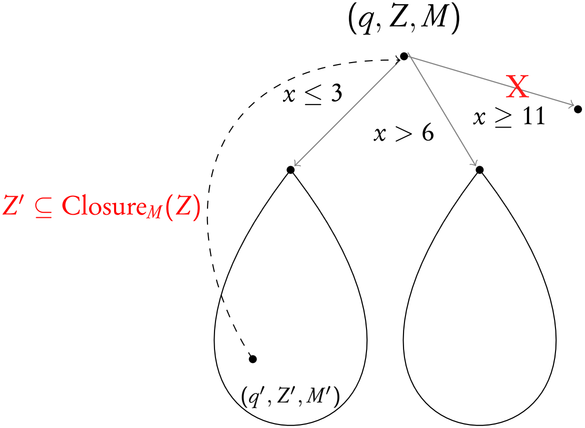
$M(x) = \quad 6$



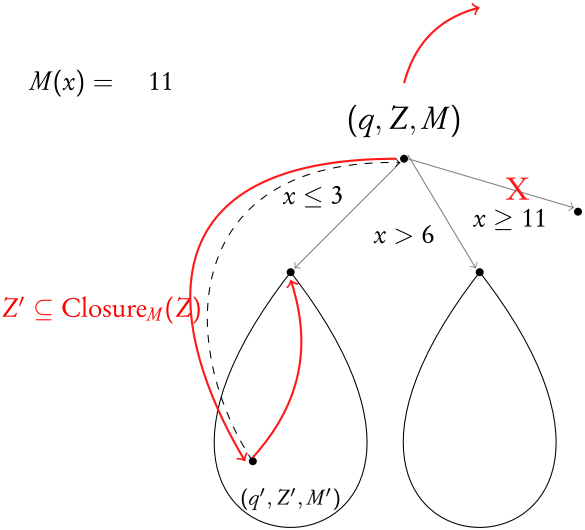
$M(x) = \quad 6$



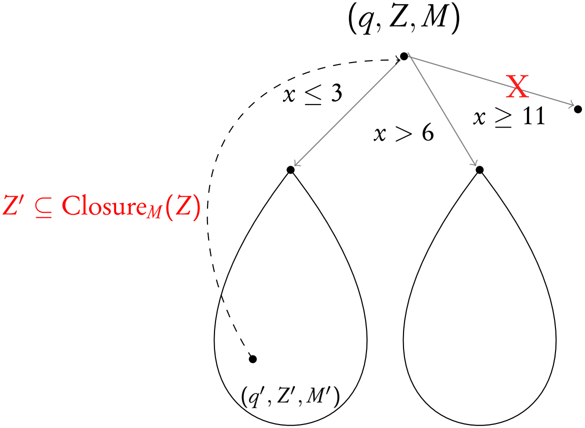
$M(x) = \quad 11$



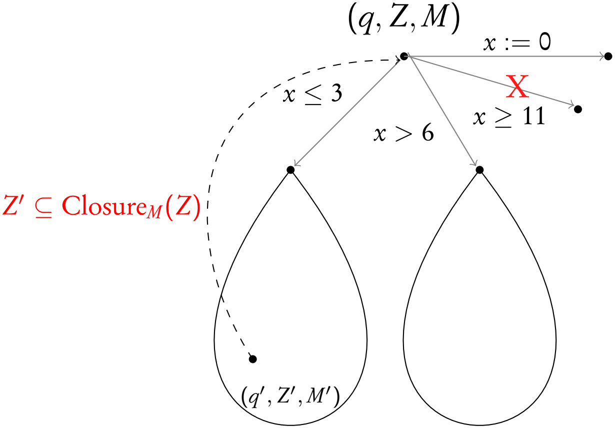
$M(x) = \quad 11$



$M(x) = \quad 11$

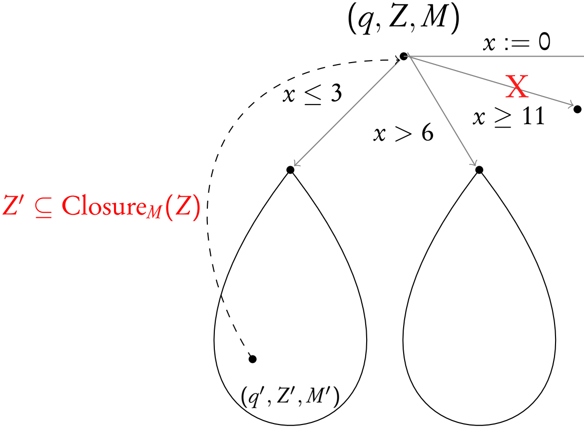


$M(x) = \quad 11$

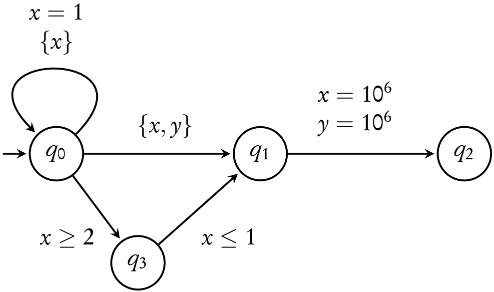


$M(x) = \quad 11$

All **tentative nodes** consistent
+ No more **exploration**
$\rightarrow$ Terminate!



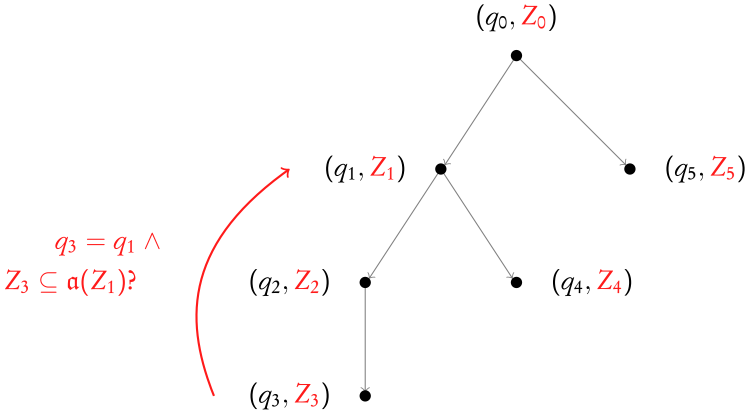
Constant propagation



Theorem (Correctness)

An accepting state is reachable in $\mathcal{A}$ iff the constant propagation algorithm reaches a node with accepting state and a non-empty zone.

Key idea: Compute the **zone graph**, use **abstraction** for termination



Developments are recent, a lot of (not-so-low) hanging fruit available

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