Causal Ambiguity and Partial Orders in Event Structures

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Abstract

Event structure models often have some constraint which ensures that for each system run it is clear what are the causal predecessors of an event (i.e. there is no causal ambiguity). In this contribution we study what happens if we remove such constraints. We define five different partial order semantics that are intentional in the sense that they refer to syntactic aspects of the model. We also define an observational partial order semantics, that derives a partial order from just the event traces.

It appears that this corresponds to the so-called early intentional semantics; the other intentional semantics cannot be observationally characterized. We study the equivalences induced by the different partial order definitions, and their interrelations.

1   Introduction

Prominent models for non-interleaving semantics are the *event structure* models. Event structures have as their basic objects labelled events together with relations representing causality and conflict. Originally event structures were used for giving a semantics to Petri nets [Win80]. They have been also used as a semantics for process algebraic languages like CCS [BC94], CSP [LG91] and LOTOS [Lan92]. Several different types of event structures exist: we mention prime event structures [Win80, Win89], stable event structures [Win89], flow event structures [BC94], and bundle event structures [Lan93, Lan92].

All these models are causally disambiguous, by which we mean the following: if an event has happened, there is exactly one set of causal predecessors of the

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event, i.e. there is never any ambiguity in deciding which are the causes of an event.

This is an important technical property, especially if one wants to relate an event structure model to the more fundamental model of partially ordered sets (or *posets*). Posets can be used as the underlying semantics of many different models; for an elaborate motivation of the importance of posets we refer to [Ren93]. Absence of causal ambiguity implies that there is exactly one poset corresponding to a system run.

Posets can be defined in two alternative ways: by referring to the causality representation in the model (we call this *intentional*), or by just referring to the system runs (we call this *observational*). Having corresponding intentional and observational characterizations of the posets is important for relating event structures to other models, where an explicit representation of causality may be absent.

In e.g. stable or bundle event structures the absence of causal ambiguity (this property is called *stability* in [Win89]) is due to a constraint on the model, which roughly says that if there are alternative causes for an event, then these causes should somehow be in conflict.

For certain application areas (e.g. business redesign) it can be argued that this constraint is too restrictive [Fer94]. Therefore the problem this paper addresses is the following: is it possible to define a partial order semantics for an event structure model with causal ambiguity ?

The organization of the paper is as follows. In section 2 we present an event structure model and sketch the problem of causal ambiguity. In section 3 we give five intentional poset definitions, and in section 4 we show that exactly one of them (the so–called early causality) has an observational characterization. In section 5 we look at the induced equivalence relations, and section 6 is for conclusions.

2 Event structures

Event structure models have as their basic ingredient events labelled with actions; an event models the occurrence of its action. Different events can have the same action label, implying that they model different occurrences of the action. Action labels do not play a role in this paper but are important when the model is used e.g. as a semantics for a language. We are in general not interested in the event identities as such (so implicitly we work modulo an event renaming morphism), as the events just serve to identify or distinguish action occurrences. Often we will denote an event by its action label, if no confusion arises.

Two events in a system are said to be in *conflict* if there is no system run in which both events happen. In this paper we will restrict ourselves to the representation of conflict by a binary relation between events. In that case the main difference between the models lies in the way they represent causality.

In prime event structures causality is modelled by a partial order on the set of

events. This model is mathematically very elegant and convenient. The drawback is that as a consequence each event has a unique enabling, so if an action can be caused in alternative ways we need to model the action by different events, harmful to the conciseness of models. In addition it may be rather complicated to define some operations on prime event structures, especially parallel synchronization.

For these reasons other models like stable, flow and bundle event structures model causality in a different way. Flow event structures model causality by a flow relation that (contrary to prime event structures) need not be transitive, thereby making it possible for an event to have alternative enablings. However, also for flow event structures parallel synchronization is a bit problematic as it is technically dependent on self-conflicting events (as we argued in [Lan92]).

The first event structure model that was defined in order to allow for multiple enablings is the model of *stable* event structures [Win89]. There causality is represented by a set ⊢ of *enablings*, which are pairs (X, e), with X a set of events and e an event, denoted by $X \vdash e$. The interpretation is that e can happen if for some enabling $X \vdash e$ all the events in X have happened already.

In this paper we use bundle event structures as our illustrative vehicle, since for them some necessary technical results are readily available. However, in the full version of this paper [LBK97] we have shown that the approach here applies to stable event structures just as well.

Since concepts, like well-foundedness [Win89], that address problems with infinite sets of events are orthogonal to the issues of this paper and need not bother us here, we conveniently restrict ourselves to finite sets of events.

2.1 Bundle event structures

In bundle event structures [Lan93, Lan92], causality is represented by *bundles*: a bundle is a pair (X, e) with X a set of events and e an event. The set of all bundles is denoted by $\mapsto$ and we denote a bundle (X, e) by $X \mapsto e$.

The meaning of a bundle $X \mapsto e$ is that X is a set of causal conditions for e, in the sense that if e happens, one of the events in X has to have happened before. If several bundles point to e, for each bundle set an event should have happened. In addition, we demand that for each bundle $X \mapsto e$, all the events in X are in mutual conflict with each other. In this way, if e has happened, exactly one event from X has happened before, so there is no doubt about which are the causal predecessors of e. In the next section we see what happens if we remove this condition.

The definition of bundle event structures:

Definition 2.1 A *bundle event structure* $\mathcal{E}$ is a 4-tuple
$\mathcal{E} = (E, \#, \mapsto, l)$ with :

- E a set of *events*

- $\# \subseteq E \times E$, the symmetric and irreflexive *conflict* relation

- $\mapsto \subseteq 2^E \times E$, the *bundle* set

- $l : E \rightarrow Act$, the *labelling* function

such that the following property holds:

P1: $X \mapsto e \Longrightarrow \forall e_1, e_2 \in X : (\ e_1 \neq e_2 \Longrightarrow e_1 \# e_2\ )$ □

We represent a bundle event structure graphically in the following way. Events are drawn as dots; near the dot we sometimes give the event name and/or the action. Conflicts are indicated by dotted lines. A bundle $X \mapsto e$ is indicated by drawing an arrow from each element of X to e and connecting all the arrows by small lines.

The following picture is an example of a bundle event structure, with a bundle $\{a, b, c\} \mapsto d$:
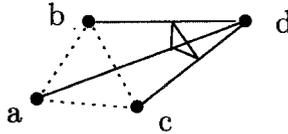
The bundle here means that for d to happen, either a, b or c should have happened already.

The concept of a system run for a bundle event structure is captured by the notion of an *event trace*, which is a conflict–free sequence of events, where each event is preceded by its causal predecessors:

Definition 2.2 Let $\mathcal{E} = (E, \#, \mapsto, l)$ be a bundle event structure. An *event trace* is a sequence of distinct events $e_1, \ldots, e_n$, with $e_1, \ldots, e_n \in E$, satisfying:

- $\{e_1, \ldots, e_n\}$ is conflict-free, i.e. $\forall e_i, e_j : \neg(e_i \# e_j)$.

- $X \mapsto e_i \Longrightarrow \{e_1, \ldots, e_{i-1}\} \cap X \neq \emptyset$

□

Notation: Let $\sigma = e_1 \ldots e_n$ be an event trace, then $\widehat{\sigma} = \{e_1, \ldots, e_n\}$ is the set of events in σ.

With the help of event traces we can define a semantics for bundle event structures in terms of (labelled) partial orders, abbreviated *posets* (not to be confused with *pomsets*, which are equivalence classes of posets modulo event renaming morphisms [Pra86]). Posets form a natural and attractive basic semantics for comparing true concurrency models [Ren93].

The next definition and theorem show how to obtain posets from event traces:

Definition 2.3 Let σ be an event trace of $\mathcal{E}$, with $\widehat{\sigma} = T$. We define the *precedence relation* $\prec_T \subseteq T \times T$ by $e \prec_T e'$ iff $\exists X \subseteq E : (\ e \in X \wedge X \mapsto e'\ )$. The relation $\leq_T$ is defined as $\leq_T = \prec_T^*$, i.e. the reflexive and transitive closure of $\prec_T$. □

Theorem 2.4 $\leq_T$ is a partial order over T.

Proof : see [Lan92] □

Let $\mathcal{E}$ be a bundle event structure, then the set of posets we get by applying definition 2.3 to all event traces of $\mathcal{E}$ is denoted by $P(\mathcal{E})$, where P stands for posets.

2.2 Observational partial orders

We have called the above definitions of partial order (obtained from an event trace) *intentional*, as opposed to *observational*, because they refer to aspects of the model, viz. bundles, that are not observable as such. Therefore the question arises how to relate these partial orders to systems where the only observations that can be made are the event traces. As an answer to this question we give a definition of partial orders from event traces that is only based on event traces and does not need to take recourse to bundles. We call this definition observational, even though a rather strong notion of observation is assumed, namely the ability to observe events (so the occurrence of actions, instead of just actions).

It is easy to prove that each event trace is a linearization of the partial order we get by definition 2.3. This provides the basic intuition for the observational poset definition, which works as follows.

Let σ be an event trace of a bundle event structure $\mathcal{E}$, with set of events $\widehat{\sigma} = T$. Now consider all event traces of $\mathcal{E}$ with the same events as σ and suppose $\{\sigma' \mid \widehat{\sigma'} = T\} = \{\sigma_1, \ldots, \sigma_m\}$.

We associate with each event trace σ_i an ordering $\leq_i$ on its events, which is simply the order of the events in the event trace, so if $\sigma_i = e_{i1} \ldots e_{in}$ then $\leq_i$ is defined by $e_{i1} \leq_i e_{i2} \leq_i \ldots \leq_i e_{in}$.

Now define $\leq_T$ by $\leq_T = \leq_1 \cap \leq_2 \cap \ldots \cap \leq_m$. It is not hard to see that $\leq_T$ is a partial order over T, so $(T, \leq_T)$ is a partially ordered set or poset.

Let $\mathcal{E}$ be a bundle event structure, then the set of posets we get by applying the above definition to all event traces of $\mathcal{E}$ is denoted by $OP(\mathcal{E})$, where OP stands for *observational* posets.

In Corollary 7.5.4. in [Lan92] it is stated that $P(\mathcal{E}) = OP(\mathcal{E})$, i.e. the intentional posets are equal to the observational posets. This correspondence between the intentional and the observational definition makes it possible to relate bundle event structures to other models that can be defined to generate event traces, e.g. Petri nets or process algebras [Lan92].

2.3 The problem of causal ambiguity

Crucial for the definitions above is the constraint P1 (see definition 2.1), that says that from each bundle only one event can happen. If we would not have constraint P1, then the following would be a "bundle" event structure:
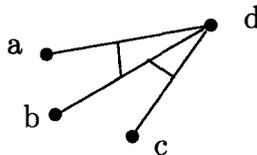
with bundles $\{a, b\} \mapsto d$ and $\{b, c\} \mapsto d$. Suppose we would take event trace *abcd* and would ask what partial order corresponds to this event trace. What are the causal predecessors of d ? With constraint P1 this question always has a unique answer, but now there are several candidates: $\{a, c\}$, $\{b\}$, $\{a, b\}$, $\{b, c\}$ and $\{a, b, c\}$ are all candidate sets of causal predecessors of d. We therefore have

to adapt our definition of how to obtain a partial order from an event trace, and in the next section we will see that there are several ways of doing so.

Also the observational definition of the previous section does not work anymore. If we try the recipe given there for the above event structures, we obtain 14 event traces with events $\{a, b, c, d\}$; the intersection of these linear orders is a poset with just the identity as the ordering relation, which surely does not capture the causality information of the event structure.

Bundle event structures without constraint P1 have been baptized *dual* event structures in [Kat96]. Providing intentional and observational partial order definitions for dual event structures is the theme of the following sections.

3   Intentional partial order definitions

In this section we present several definitions of causality in possibly causally ambiguous situations. What definition is appropriate depends on considerations coming from the application area. In this respect the situation is very similar to the field of implementation relations [vG90], where many different implementation relations exist, each with its own (often observational) justification. In fact in section 5 we show how these different causality notions give rise to different partial order equivalences, and study their interrelations. In section 4 we show that only one of the notions in this section has an observational characterization in terms of event traces.

By a *cause* of e in σ we mean a set of causal predecessors of e, that is a set of events that enable e to happen. Each of the notions in this section gives an answer to the following question: suppose we have a dual event structure $\mathcal{E}$, with an event trace σ, and an event e in σ, what are the possible causes C in σ of e ? We do not demand that C is always unique, i.e. in principle we allow a set $\{C_i\}$ of possible causes as an answer to our question (some notions lead to a unique C though).

We can define partial orders on $\hat{\sigma}$ in the following way: for each e in σ, choose a cause C_e. Now define for all $e, e' \in \hat{\sigma}$: $e' \prec e$ iff $e' \in C_e$ and define the ordering relation on $\hat{\sigma}$ to be the transitive and reflexive closure of $\prec$. If each cause C_e occurs before e in σ (and all notions we consider have this property, in agreement with the common sense idea that causes have to occur before effects) it is easy to see that this definition leads indeed to a partial order.

3.1   Liberal causality

The least restrictive notion of causality, which we call the *liberal* one, is the one saying that each set of events from bundles pointing to e that satisfies all bundles is a cause.

Definition 3.1 *Liberal:* Let σ be an event trace of $\mathcal{E}$, e an event in this trace, and all bundles pointing to e given by $X_1 \mapsto e, \ldots, X_n \mapsto e$.
A set C is a cause of e in σ iff the following conditions hold:

- each $e' \in C$ occurs before e in σ

- $C \subseteq X_1 \cup \ldots \cup X_n$

- for all i: $X_i \cap C \neq \emptyset$

The set of posets obtained in this way from σ is denoted by $P_{lib}(\sigma)$ □

Example 3.2 Consider event trace $abcd$ of event structure
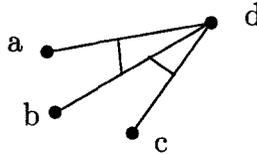
Then $P_{lib}(abcd)$ consists of the posets

□
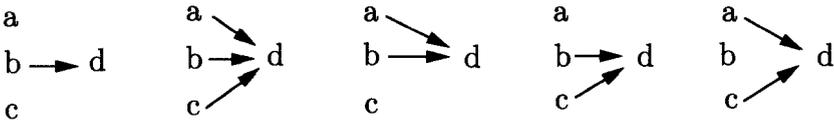
3.2 Bundle satisfaction causality

This causality notion is based on the idea that for an e in σ each bundle pointing to e is satisfied by exactly one event in a cause of e. This means that for all bundles pointing to e, each bundle can be mapped to an event in a cause C such that all events in C are being mapped upon, so the presence of each event e' in C should be justified by some bundle $X \mapsto e$, with $e' \in X$, that is associated to e'.

Definition 3.3 *Bundle satisfaction:* Let σ be an event trace of $\mathcal{E}$, e an event in this trace, and all bundles pointing to e given by $X_1 \mapsto e, \ldots, X_n \mapsto e$.
A set C is a cause of e in σ iff the following conditions hold:

- each $e' \in C$ occurs before e in σ

- There is a surjective mapping $f : \{X_i\} \to C$ such that $f(X_i) \in X_i$

The set of posets obtained in this way from σ is denoted by $P_{bsat}(\sigma)$ □

Example 3.4 Let $\mathcal{E}$ be the same dual event structure as in example 3.2. Now we allow e.g.
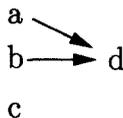
(where a satisfies bundle $\{a, b\} \mapsto d$ and b satisfies bundle $\{b, c\} \mapsto d$) and

$$a$$
$$b \longrightarrow d$$
$$c$$
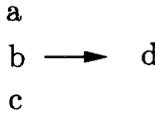
(where b satisfies both bundles $\{a, b\} \mapsto d$ and $\{b, c\} \mapsto d$). Notice that we do allow more events from one bundle, or several bundles satisfied by the same event.

$$a \searrow$$
$$b \rightarrow\!\!\!\rightarrow d$$
$$c \nearrow$$
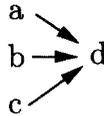
is not allowed as a poset, as d has three causal predecessors and there are only two bundles to be satisfied. $\square$

Clearly each C satisfying definition 3.3 also satisfies definition 3.1, so for all event traces σ, $P_{bsat}(\sigma) \subseteq P_{lib}(\sigma)$.

3.3 Minimal causality

The next causality definition is based on the idea that each cause should be minimal, in the sense that there is no subset which is also a cause.

Definition 3.5 *Minimal:* Let σ be an event trace of $\mathcal{E}$, e an event in this trace, and all bundles pointing to e given by $X_1 \mapsto e, \ldots, X_n \mapsto e$.
A set C is a cause of e in σ iff the following conditions hold:

- each $e' \in C$ occurs before e in σ

- for all i: $X_i \cap C \neq \emptyset$

- there is no proper subset of C satisfying the previous two conditions

The set of posets obtained in this way from σ is denoted by $P_{min}(\sigma)$ $\square$

Example 3.6 Let $\mathcal{E}$ be the same dual event structure as in example 3.2. Now the only posets for trace $abcd$ are

$$a \searrow$$
$$b \quad\searrow d \qquad \text{and} \qquad a$$
$$c \nearrow \qquad\qquad\qquad b \longrightarrow d$$
$$\qquad\qquad\qquad\qquad\qquad c$$

E.g.

$$a \searrow$$
$$b \longrightarrow\!\!\!\rightarrow d$$
$$c$$
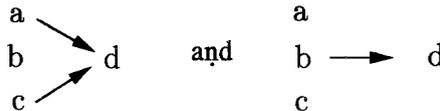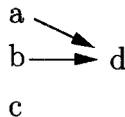
is not allowed anymore as $\{a, b\}$ is not minimal: also the subset $\{b\}$ would be sufficient for d to be enabled. $\square$

Again it is easy to see that each C satisfying definition 3.5 also satisfies definition 3.3, so for all event traces σ, $P_{min}(\sigma) \subseteq P_{bsat}(\sigma)$.

3.4 Early causality

If one is trying to remove "superfluous" events from the causes, at first sight the minimal definition given above seems hard to improve upon. However, look at the following example.

Example 3.7 Consider trace abc from event structure
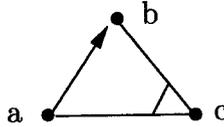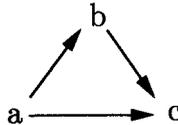
(with $a \leq c$ because of transitivity). However, if b happens, a has happened already, and a is enough to let c happen. So in a sense the causality relation between b and c is superfluous. □

In order to remove this superfluousness, we would like to demand that a cause is somehow the "earliest".

Definition 3.8 Let $\sigma = e_1 \ldots e_n$ be an event trace, and let $C, C' \subseteq \{e_1 \ldots e_n\}$. We say C is earlier than C', notation $C \ll C'$, iff the maximal index in σ of the events in $C \setminus C'$ is smaller than the maximal index in σ of the events in $C' \setminus C$ (we define the maximal index of $\emptyset$ to be 0). □

Lemma 3.9 Let σ be an event trace, let Id be the identity relation over all the subsets of $\widehat{\sigma}$. The relation $\ll \cup Id$ is a total order over all the subsets of $\widehat{\sigma}$.

Proof : Represent a subset C of $\widehat{\sigma}$ by a binary n-digit, where the ith digit is 1 iff $e_i \in C$, the nth digit being the most significant one. Call the resulting number $n(C)$, then it is easy to see that $C \ll C'$ iff $n(C) < n(C')$. □

Given a set of subsets of $\widehat{\sigma}$, lemma 3.9 ensures that it makes sense to talk of a unique earliest element of this set. Now we are ready for the definition of early causality:

Definition 3.10 *Early:* Let σ be an event trace of $\mathcal{E}$, e an event in this trace, and all bundles pointing to e given by $X_1 \mapsto e, \ldots, X_n \mapsto e$.
A set C is a cause of e in σ iff the following conditions hold:

- each $e' \in C$ occurs before e in σ

- for all i: $X_i \cap C \neq \emptyset$

- C is the earliest set satisfying the previous two conditions.

The set of posets obtained in this way from σ is denoted by $P_{early}(\sigma)$ □

Note that due to the uniqueness of the earliest enabling, this definition leads to a unique cause in an event trace σ, and so to a unique poset for σ.

It is easy to check that if $C \subset C'$ then $C \ll C'$; this means that each earliest cause C is also minimal, so for all event traces σ, $P_{early}(\sigma) \subseteq P_{min}(\sigma)$.

3.5   Late causality

In the last section we defined an early causality, taking always the earliest cause. One might ask if it would also be possible to ask for the latest possible cause. Think for instance of a situation where events write values into variables; then it would be natural to consider the last write as a causal predecessor of e.g. an event that reads the variable.

We define C later C' iff $C' \ll C$. Now it is not the case that latest implies minimality (on the contrary, a superset of a set C will always be later). Therefore in the definition of late causality we have to explicitly state that the cause is a minimal one, whereas for early causality this was a consequence.

Definition 3.11 *Late:* Let σ be an event trace of $\mathcal{E}$, e an event in this trace, and all bundles pointing to e given by $X_1 \mapsto e, \ldots, X_n \mapsto e$.
A set C is a cause of e in σ iff the following conditions hold:

- each $e' \in C$ occurs before e in σ

- for all i: $X_i \cap C \neq \emptyset$

- there is no proper subset of C satisfying the previous two conditions

- C is the latest set satisfying the previous three conditions

The set of posets obtained in this way from σ is denoted by $P_{late}(\sigma)$ □

Each C satisfying definition 3.11 trivially satisfies definition 3.5, so for all event traces σ, $P_{late}(\sigma) \subseteq P_{min}(\sigma)$.

3.6   Comparisons

We saw that for each event trace σ, $P_{late}(\sigma), P_{early}(\sigma) \subseteq P_{min}(\sigma) \subseteq P_{bsat}(\sigma) \subseteq P_{lib}(\sigma)$.

We can extend the definition of P_x to dual event structures by having $P_x(\mathcal{E})$ denote the posets of all event traces of event structure $\mathcal{E}$.

The subset inclusions for the posets of a single event trace carry over to the subset relations for the posets of a dual event structure. These inclusions are strict because of the following reasons. For $\mathcal{E}$ in example 3.2 we have seen that $P_{min}(\mathcal{E}) \subset P_{bsat}(\mathcal{E}) \subset P_{lib}(\mathcal{E})$. For $\mathcal{E}$ in example 3.7 we have that $P_{early}(\mathcal{E}) \subset P_{min}(\mathcal{E})$. An example of a dual even structure that has a minimal poset that is not a late poset: let $\mathcal{E}$ be the following dual event structure:
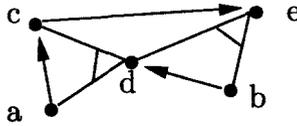
We invite the reader to check that
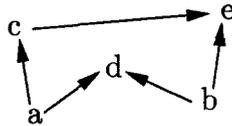
is a minimal poset for e.g. event trace *abcde*, but cannot be a late poset for any event trace of $\mathcal{E}$.

4 An observational partial order definition

We would like to have also for the dual or instable event structure an observational definition of partial order like the one in section 2.3 (cf. definition 2.3). As illustrated in section 3, we cannot use the technique of reconstructing the posets from their linearizations (the event traces) as we end up with posets that have too little ordering and do not model the causality in a satisfactory way. We therefore try another recipe.

The idea of this definition is the following: for an event e in σ, we look at all event traces with the same events as σ. We then look at the set of predecessors of e in some event trace (we call such a set a *securing* for e). From all these securings we now take the earliest securing for e in σ and define $e' \leq e$ for all e' in this earliest securing.

Definition 4.1 Let σ be an event trace of a dual event structure $\mathcal{E}$, and e an event in σ.

- let $[\sigma]$ be the set of all event traces of $\mathcal{E}$ with events $\hat{\sigma}$

- the securings of e are defined as $\{\widehat{\sigma_1} | \exists \sigma_2 : \sigma_1 e \sigma_2 \in [\sigma]\}$

- take the earliest securing S in σ and define $e' \leq e$ iff $e' \in S \cup \{e\}$

□

The nice result is that $\leq$ as defined by the observational definition 4.1 is exactly the unique partial order as defined by the intentional one of *early* causality.
Let $\leq$ be the ordering defined by definition 4.1, then we write $OP(\sigma)$ (for *observational poset*) for $(\hat{\sigma}, \leq)$.

Theorem 4.2 Let σ be an event trace of dual event structure $\mathcal{E}$. Then:
$OP(\sigma) = P_{early}(\sigma)$.

Proof : See [LBK97]   □

So early causality can also be characterized in an observational way. Is it possible to find a characterization for any of the other intentional causality concepts ? The answer is no, as can be learned from the following example.

Example 4.3 The dual event structures
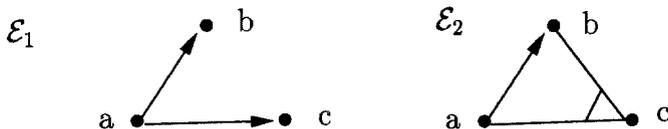
have the same event traces. $\mathcal{E}_2$ has for trace abc the poset
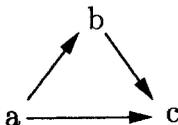
under liberal, bundle satisfaction, minimal and late causality, but this is not a poset of $\mathcal{E}_1$.   □

Any observational definition of causality would have the same result for $\mathcal{E}_1$ and $\mathcal{E}_2$ above as they have the same traces. Since the other intentional causality concepts lead to different posets for $\mathcal{E}_1$ and $\mathcal{E}_2$ this shows that these intentional concepts cannot be observationally characterized.

So the result is that the early causality concept is the only one that can be observationally characterized.

5   Partial order equivalence relations

The causality notions defined in the previous sections induce equivalence relations in the following way:

Definition 5.1 Let $\mathcal{E}_1$, $\mathcal{E}_2$ be dual event structures. We define $\mathcal{E}_1 \approx_x \mathcal{E}_2$ iff $P_x(\mathcal{E}_1) = P_x(\mathcal{E}_2)$, where $x \in \{lib, bsat, min, early, late\}$.   □

Now an obvious question is the relation between the different equivalence relations. First of all, we note that due to theorem 4.2, $\approx_{early}$ is equal to event trace equivalence (since equal event traces lead to the same observational posets so to the same early posets, and vice versa). We have the following two implications:

Theorem 5.2

   1. $\mathcal{E}_1 \approx_x \mathcal{E}_2 \implies \mathcal{E}_1 \approx_{early} \mathcal{E}_2$ for $x \in \{lib, bsat, min, late\}$

   2. $\mathcal{E}_1 \approx_{bsat} \mathcal{E}_2 \implies \mathcal{E}_1 \approx_{lib} \mathcal{E}_2$

Proof : See [LBK97] □

The two above implications are strict (i.e. the reverse does not hold). Moreover, no other implications hold. This can be seen from the following examples, where each pair of dual event structures is event trace equivalent and so early equivalent:

Example 5.3

$\mathcal{E}_1$ •b

$\not\approx_{lib}$
$\not\approx_{bsat}$
$\not\approx_{min}$
$\not\approx_{late}$

$\mathcal{E}_2$
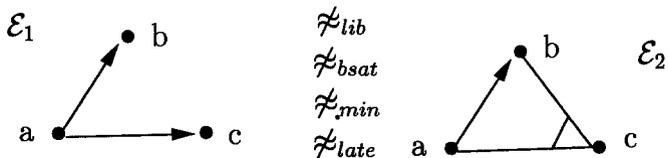
as $\mathcal{E}_2$ can have $b \leq c$ and $\mathcal{E}_1$ can not. □

Example 5.4

$\mathcal{E}_1$ •c

$\not\approx_{lib}$
$\not\approx_{bsat}$
$\approx_{min}$
$\approx_{late}$

$\mathcal{E}_2$

as $\mathcal{E}_2$ can have $b \leq c$ in liberal and bundle satisfaction posets and $\mathcal{E}_1$ can not. For minimal and late causality, b will not be in a cause for c as a is sufficient. □

Example 5.5

$\mathcal{E}_1$

$\approx_{lib}$
$\not\approx_{bsat}$
$\approx_{min}$
$\approx_{late}$

$\mathcal{E}_2$
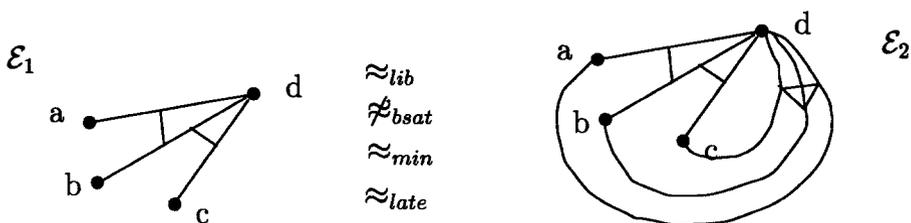
as the extra bundle $\{a, b, c\} \mapsto d$ has no influence on liberal, minimal and late causes, but $P_{bsat}(\mathcal{E}_2)$ has poset

$$a \searrow$$
$$b \rightarrow d$$
$$c \nearrow$$
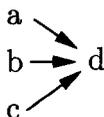
and $P_{bsat}(\mathcal{E}_1)$ has not. □

Example 5.6
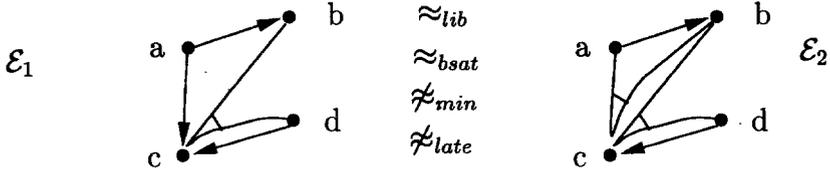
For minimal and late causality, $\mathcal{E}_2$ has poset
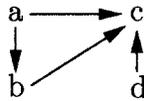
as $\{b, d\}$ is a minimal cause for c in e.g. trace $abdc$, which does not hold for $\mathcal{E}_1$.

□

The only relationship we have not been able to clear up is between $\approx_{min}$ and $\approx_{late}$. We have not been able to produce an example of their difference, nor have we been able to prove that such an example does not exist.

If we leave that relation as an open question, we can resume our findings in the following diagram:
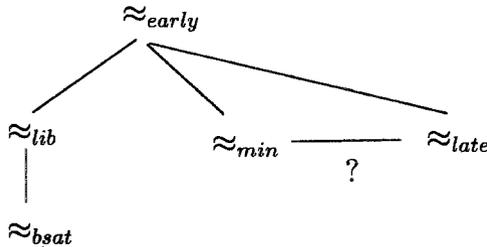
6 Conclusion

We have shown that it is possible to give a partial order semantics for a causally ambiguous event structure model. We have presented five intentional causality concepts (that make use of the way causality is represented in the model): liberal, bundle satisfaction, minimal, early and late causality. We have given an observational characterization (that makes use of just event traces) of one of them, namely the *early* causality, and have shown that for the other notions no observational characterization can be given.

Especially the fact that late causality, which at first sight seems a symmetric counterpart to early causality, cannot be observationally characterized is something that we did not expect beforehand.

We studied the induced equivalence relations and found that all equivalences imply early equivalence (which is equal to event trace equivalence), and that bundle satisfaction equivalence implies liberal equivalence.

We gave examples showing that apart from these implications the different equivalences are incomparable, except for the relation between minimal and late equivalence: the relation between these equivalences is an open question.

Another problem for further study would be to look at transformation laws preserving the various equivalences, in a similar way as has been done in [Lan92] for event trace equivalence.

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