Comparing Different Projection Operators in the Cylindrical Algebraic Decomposition for SMT Solving

Theory of Hybrid Systems Informatik 2

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SC^2 Workshop Juli 29th

Outline

Nonlinear arithmetic QF_NRA

Definition (Nonlinear arithmetic)

Boolean combinations of **polynomial** constraints over reals

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Nonlinear arithmetic QF_NRA

Definition (Nonlinear arithmetic)

Boolean combinations of **polynomial** constraints over reals

Example

$$\exists x, y. \qquad x^2 + y^2 - 4 \leq 0 \wedge (x^2 - y + 0.5 < 0 \vee x^2 + 5 \cdot y + 5 < 0)$$

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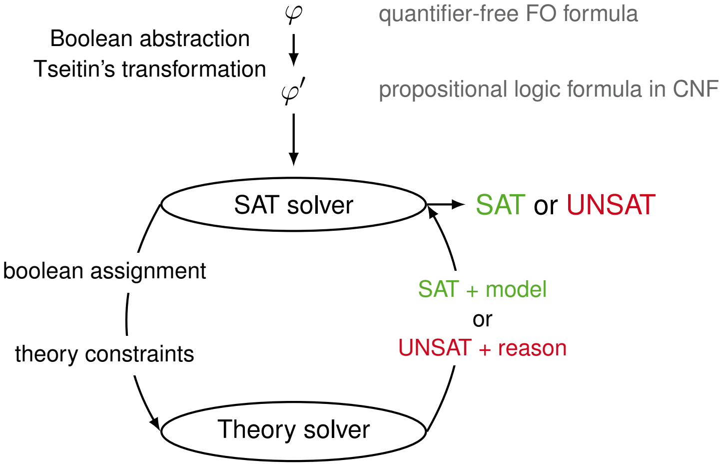
Also:

- With quantifiers (NRA)
- Over integers (QF_NIA)

SMT Solving


φ
quantifier-free FO formula
Boolean abstraction
Tseitin's transformation
φ'
propositional logic formula in CNF
SAT solver
SAT or UNSAT
boolean assignment
theory constraints
SAT + model
or
UNSAT + reason
Theory solver

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Cylindrical Algebraic Decomposition

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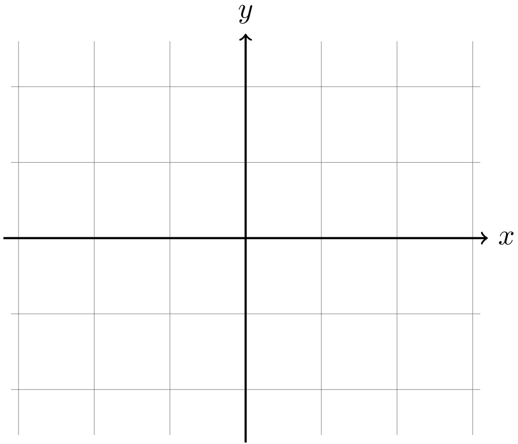

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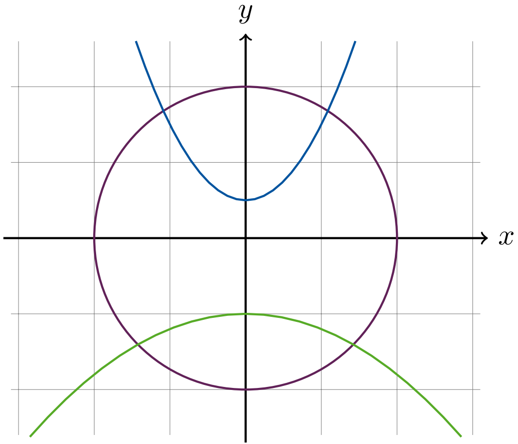

- Where are solutions?

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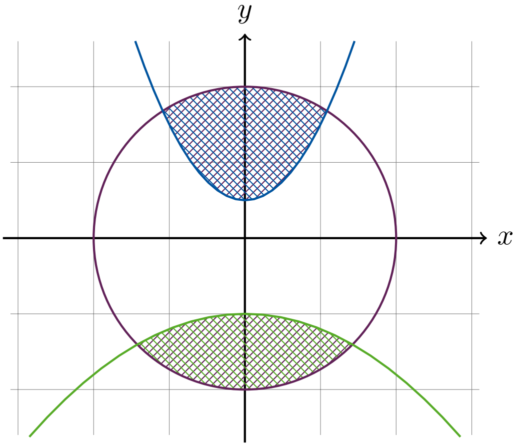

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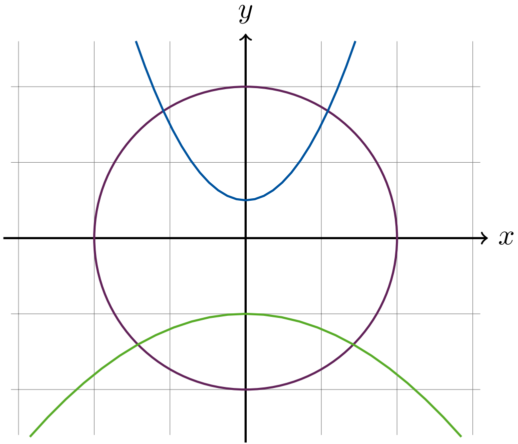

- Where are solutions?
- What would a human do?

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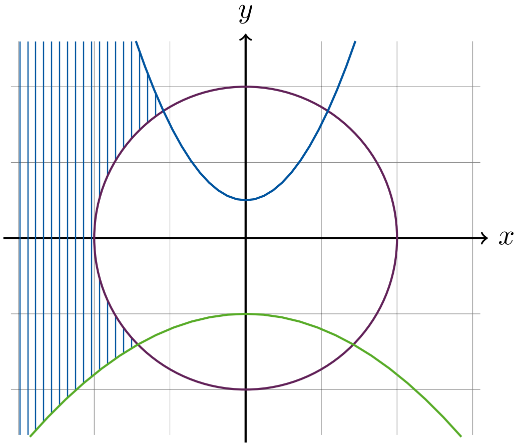

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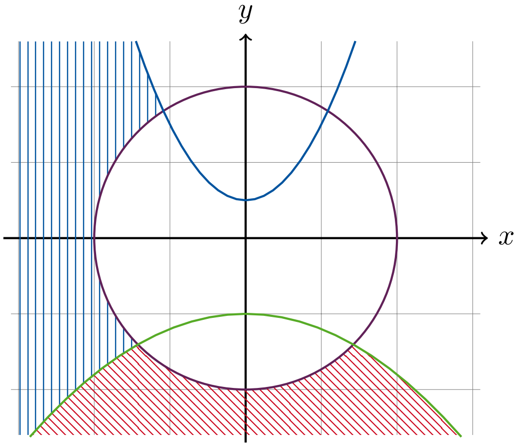

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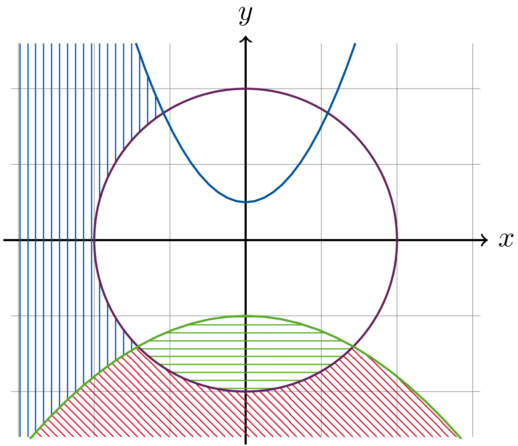

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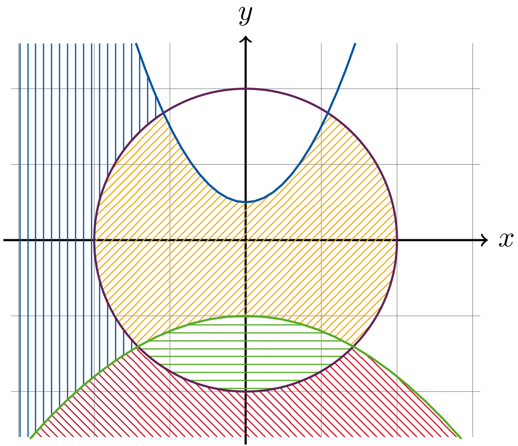

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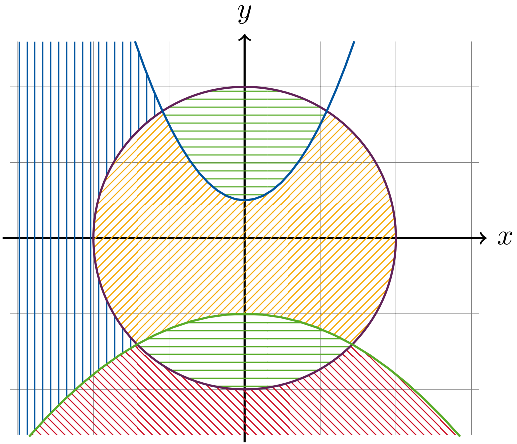

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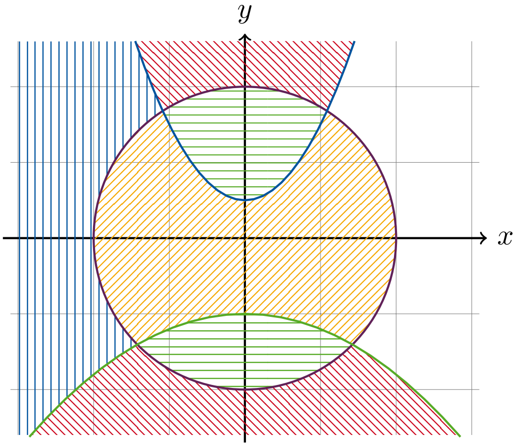

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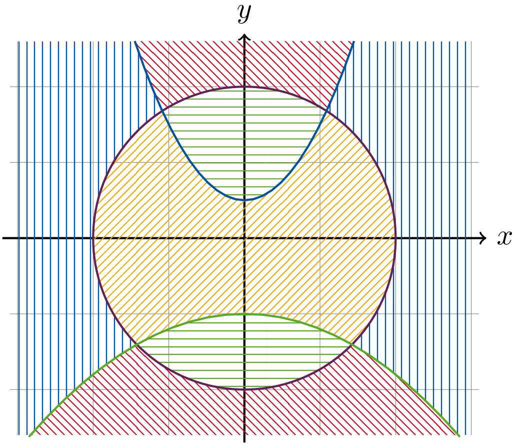

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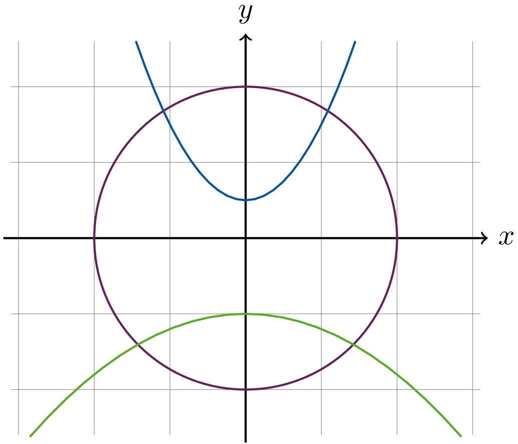

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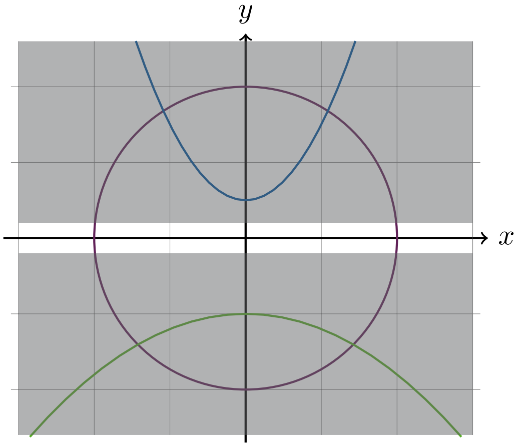

- Where are solutions?
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- What would CAD do?
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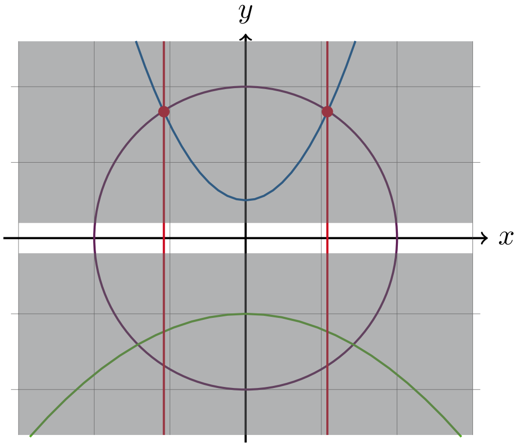

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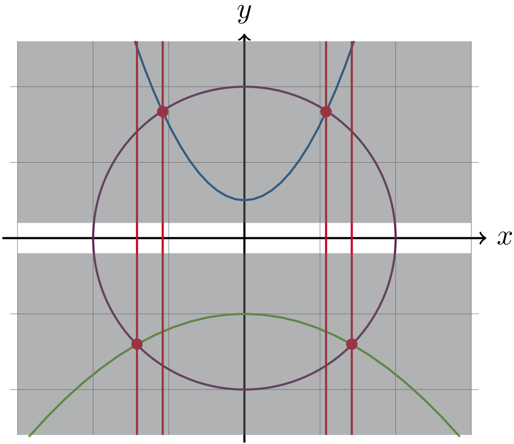

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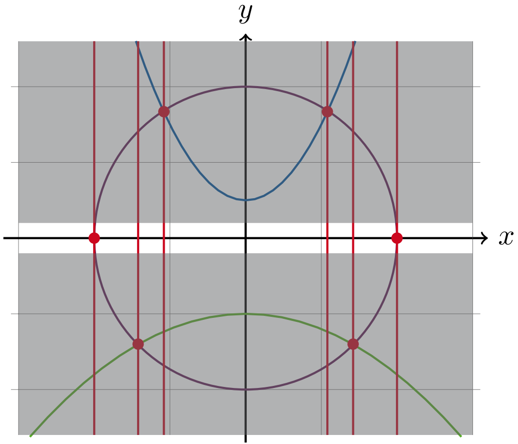

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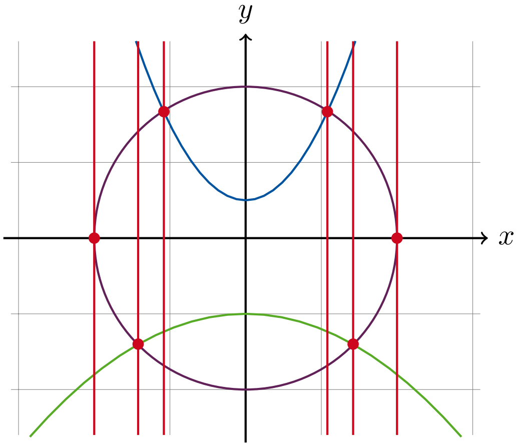

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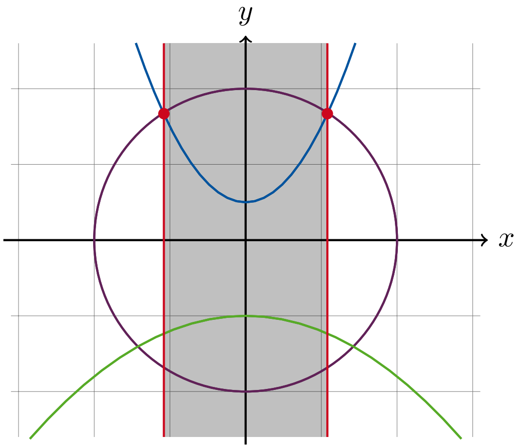

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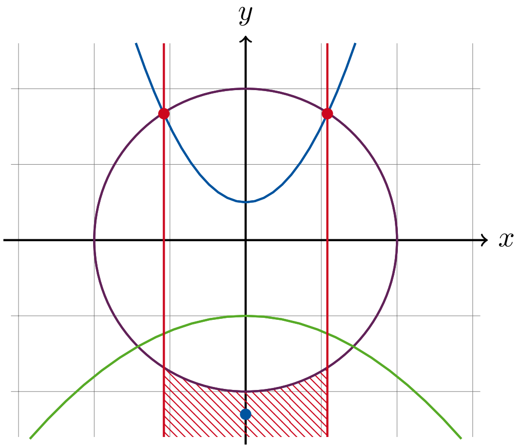

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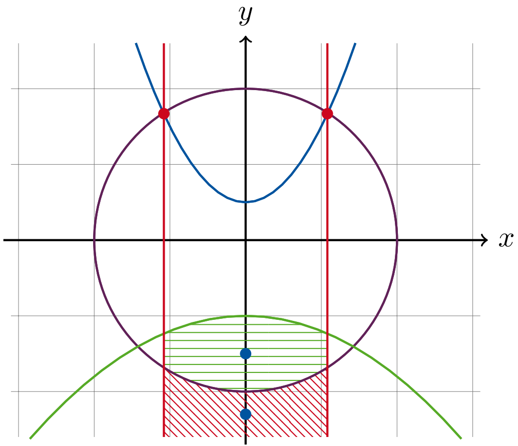

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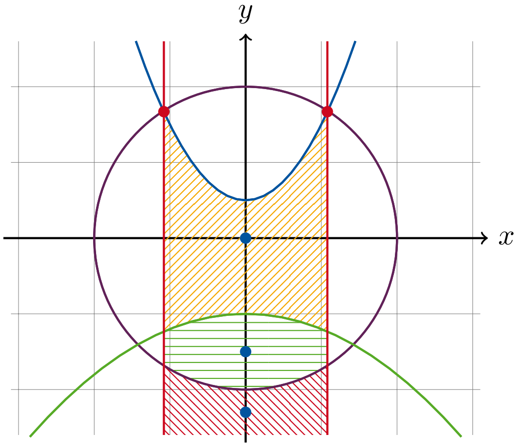

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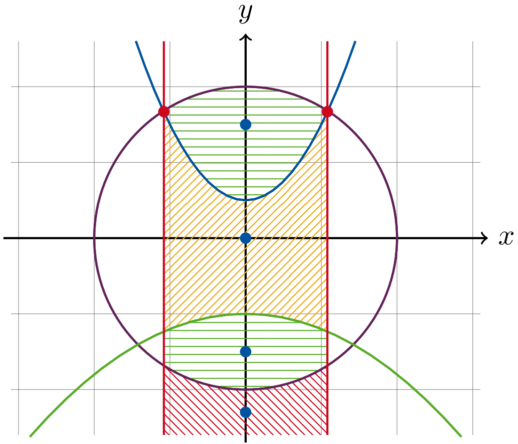

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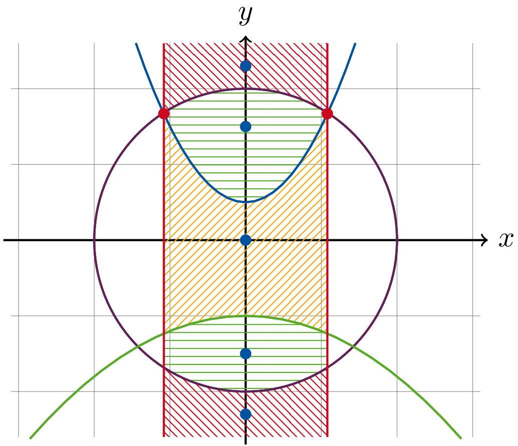

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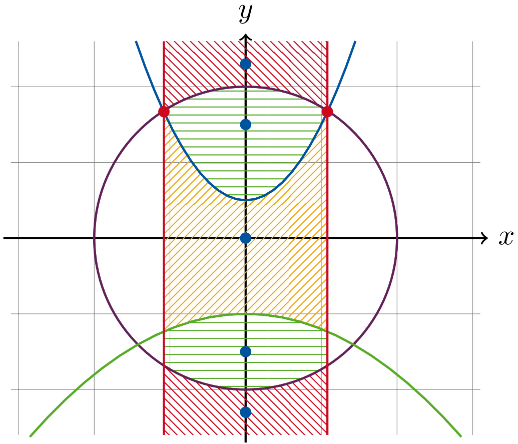


- Where are solutions?
- What would a human do?
- What would CAD do?
  - First dimension x
  - Second dimension y
  - Test sample points

Cylindrical Algebraic Decomposition

$$P_n \subseteq \mathbb{Z}[x_1, \ldots, x_n]$$

$$\Big\downarrow \text{project}$$

$$P_{n-1} \subseteq \mathbb{Z}[x_1, \ldots, x_{n-1}]$$

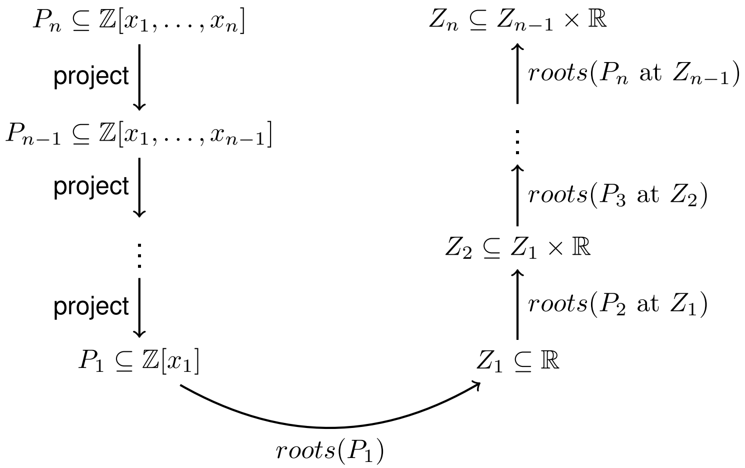
$$\Big\downarrow \text{project}$$

$$\vdots$$

$$\Big\downarrow \text{project}$$

$$P_1 \subseteq \mathbb{Z}[x_1]$$

Cylindrical Algebraic Decomposition



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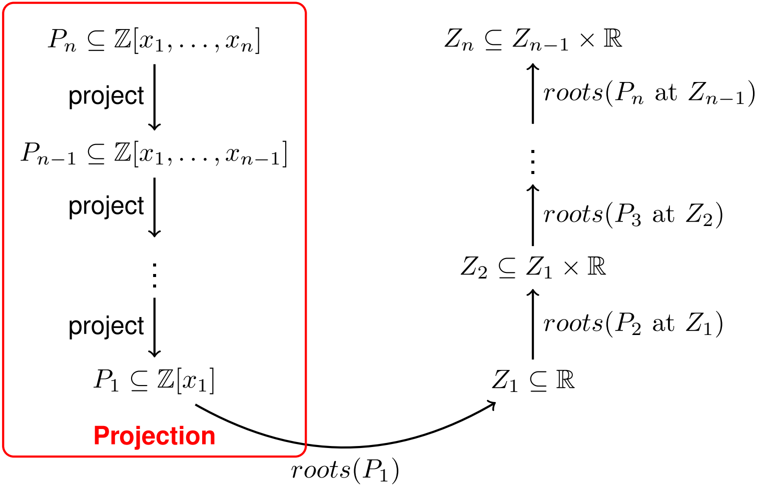

Projection Operators

Intuition

Cylinders in $\mathbb{R}^n$ based on the roots of P_{n-1} form proper stacks.
Substitute a sample from $\mathbb{R}^{n-1}$ into P_n, the roots cover all cylinders.

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We consider:

- Collins
- Hong
- McCallum
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Not considered or specific use case:

- Lazard (improvement of McCallum)
- Seidl & Sturm (based on Hong for partial CAD)
- Strzeboński („local projection“)
- Brown & Košta („OneCell CAD“)
- ...

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Notation

Definition (Polynomials)

$p = \sum_{i=0}^{m} a_i \cdot x_n^i$ in **main variable** x_n and $a_i \in \mathbb{R}[x_1, ..., x_{n-1}]$.

Definition (Simple properties)

$$coeffs(p) := \{a_0, ..., a_m\} \qquad lcf(p) := a_m$$

$$red_k(p) := \sum_{i=0}^{m-k} a_i \cdot x_n^i \qquad red(p) := \{red_k(p) \mid k = 0 \dots m\}$$

Building blocks

$$
Syl(p,q) := \begin{pmatrix}
a_k & \cdots & a_0 & & & \\
 & a_k & \cdots & a_0 & & \\
 & & \ddots & & \ddots & \\
 & & a_k & \cdots & & a_0 \\
b_l & \cdots & b_0 & & & \\
 & b_l & \cdots & b_0 & & \\
 & & \ddots & & \ddots & \\
 & & b_l & \cdots & & b_0
\end{pmatrix}
\begin{matrix} \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} l \\ \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} k \end{matrix}
$$

Building blocks

$$M_j(p,q) := \begin{pmatrix} a_k & \cdots & a_0 & & & \\ & a_k & & \cdots & a_0 & \\ & & \ddots & & & \ddots \\ & & & a_k & \cdots & a_0 \\ b_l & \cdots & b_0 & & & \\ & b_l & & \cdots & b_0 & \\ & & \ddots & & & \ddots \\ & & & b_l & \cdots & b_0 \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} l-j \\ \left.\vphantom{\begin{matrix} b \\ b \\ b \\ b \end{matrix}}\right\} k-j \end{matrix}$$

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$$

Definition (Principal subresultant coefficients)

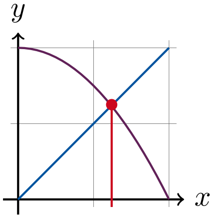
$$pcs_i(p,q) := \det(M_i)$$
$$PCS(p,q) := \{pcs_i \mid i = 0 \ldots \min(k,l)\}$$

Building blocks

Definition (Resultant)

$$res(p, q) := \det(Syl(p, q))$$

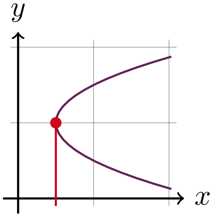
p, q have a **common root** $\Leftrightarrow$ $res(p, q)$ has a root



Definition (Discriminant)

$$disc(p) := res(p, p')$$

p has a **multiple root** $\Leftrightarrow$ $disc(p)$ has a root



Collins & Hong

Definition (Collins' operator / Hong's operator)

$$proj_C^1 := \bigcup_{p \in P} \bigcup_{r \in red(p)} \{ldcf(r)\} \cup PSC(r, r')$$

$$proj_C^2 := \bigcup_{p,q \in P} \bigcup_{\substack{r_p \in red(p) \\ r_q \in red(q)}} PSC(r_p, r_q)$$

$$proj_C := proj_C^1 \cup proj_C^2$$

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$$proj_H^2 := \bigcup_{p,q \in P} \bigcup_{r_p \in red(p)} PSC(r_p, q)$$

$$proj_H := proj_C^1 \cup proj_H^2$$

McCallum & Brown

Definition (McCallum's operator / Brown's operator)

Let P be a squarefree basis.

$$proj_M^1 := \bigcup_{p \in P} \{disc(p)\} \cup coeffs(p)$$

$$proj_M^2 := \bigcup_{p,q \in P} \{res(p,q)\}$$

$$proj_M := proj_M^1 \cup proj_M^2$$

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$$proj_M^2 := \bigcup_{p,q \in P} \{res(p,q)\}$$

$$proj_B := proj_B^1 \cup proj_M^2$$

Incomplete!

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Experiments

- SMT-RAT
  - Projections: SAT + CAD
  - Solving: SAT + VS + CAD
  - No squarefree basis, no delineating polynomials (McCallum), no additional points (Brown)
  - But: fully incremental, early abort

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- Analyzed:
  - Different projection operators
  - Different projection orders
- Not analyzed:
  - Different variable orderings
  - Different lifting orders

Projection sizes

- Project **all polynomials**, ignore boolean structure
- 5698 benchmarks where all projections terminated

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- On average 6.4 **polynomials** of degree 5.2 (**total degree** 6.1)
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	Level 1	Level 2	Level 3	Level 4
Collins	10.9 / 7.8	783.1 / 26.4	117.0 / 11.9	15.6 / 5.3
Hong	8.6 / 7.8	158.8 / 26.2	20.2 / 11.7	10.3 / 5.1
McCallum	6.1 / 6.7	16.7 / 13.3	5.1 / 5.3	7.9 / 3.8
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- **Hong** improves a lot upon Collins
- **McCallum** improves a lot upon Hong
- **Brown** improves a bit, but more speedups in lifting phase
- Hong may be viable if **incompleteness** of McCallum is an issue

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SMT solving performance

- 5698 benchmarks from before
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Operator	Solved	Timeout	average
Collins	5041	657	≈ 452.80
Hong	5125	573	≈ 233.30
McCallum	5284	414	≈ 216.38
Brown	5299	399	≈ 220.11

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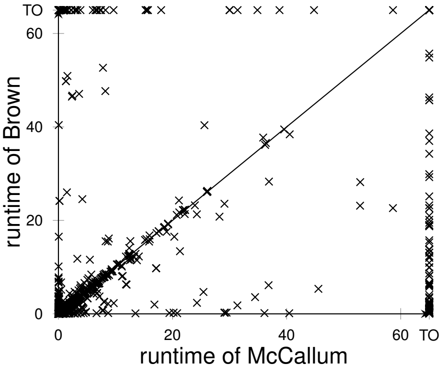

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- $\Rightarrow$ not a pressing issue **on our SMT benchmarks**

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- $\Rightarrow$ usually detrimental, sometimes essential

  required for correctness!

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- Adapt variable ordering?
- Effects of delineating polynomials and additional points?