

# Tweaking The Odds

— Parameter Synthesis in Markov Models (Abstract) —

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Markov decision processes (MDPs) are the prime model in sequential decision making under uncertainty. Their transition probabilities are fixed. Transitions in *parametric* Markov models are equipped with functions (e.g., polynomials or rational functions) over a finite set of parameters  $x_1$  through  $x_n$ , say. Instantiating each variable  $x_i$  with a value  $v_i$  induces a MDP or a Markov chain (MC) if non-determinism is absent. We present recent advances on the *parameter synthesis problem*: for which parameter values — and for MDPs, for which policy — does the instantiated Markov model satisfy a given objective? For objectives such as reachability probabilities and expected costs we consider (1) an *exact* procedure and (2) an *approximative* technique. Both approaches come with a CEGAR-like procedure to obtain a good coverage of the parameter space indicating which parameter regions satisfy the property and which ones do not.

The exact approach first obtains symbolic representations of the synthesis problem at hand. This can be done using e.g., Gaussian elimination or a technique introduced by Daws at ICTAC 2004 [4] that is based on an automata-to-regular expression conversion. These symbolic representations (in fact, rational functions in  $x_1$  through  $x_n$ ) can be solved using satisfiability-modulo-theory techniques over non-linear real arithmetic [7]. This technique is applicable to parametric MCs only but extendible to conditional reachability objectives too. Using advanced reduction and implementation techniques [5] it is practically applicable to MCs of up to a few million states and 2-3 parameters.

The approximative approach removes parameter dependencies at the expense of adding new parameters and then replaces them by lower and upper bounds [9]. It reduces parameter synthesis to standard model checking of non-parametric Markov models that have one extra degree of non-determinism. Its beauty is the simplicity and applicability to both MCs and MDPs. It is applicable to models of up to about ten million states and 4-5 parameters.

Finally, we treat parameter synthesis for (3) *multiple objectives* for parametric MDPs. Whereas multi-objective model-checking of MDPs can be cast as a linear programming problem [6], its analogue for parametric MDPs results in a non-linear programming (NLP) problem. An approximate solution of this NLP problem can be obtained in polynomial time using geometric programming [3]. This technique is extendible to richer objectives such as weighted combinations of single objectives. Initial experiments indicate that this approach seems scalable to models with tens of parameters.

Parameter synthesis has abundant applications. These include *model repair* [2, 8] — how to adapt certain probabilities in a Markov model that refutes a

given objective such that the tweaked model satisfies it — and finding *minimal recovery times for randomized self-stabilizing algorithms* [1].

## References

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