

# Optimization of Model Checking by Large Block Encoding

Thomas Mertens

24.09.2014

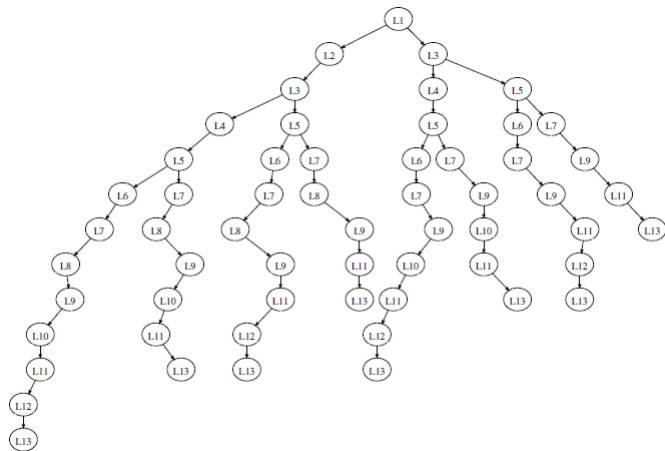
- 1 Motivation
- 2 Background
  - Program and Control-Flow Automaton
  - Model Checker
- 3 Large-Block Encoding
  - Summarization
  - Post-processing
- 4 Evaluation
- 5 Conclusion

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Why do we do Model Checking?

## Why do we do Model Checking?

- Reduce runtime and memory of model checking in particular CEGAR through minimizing the abstract reachability tree (ART)



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## Definition (Program)

A sequential program  $P$  with the operations  $Ops$  is defined as  $P = op_0, op_1, \dots, op_n$  with  $op_i \in Ops$ .

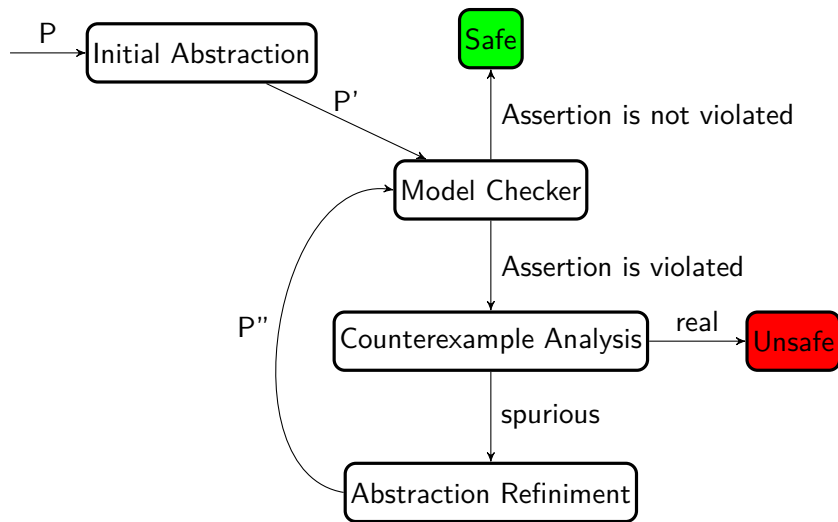
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## Definition (Control-Flow Automaton)

A Control Flow Automaton is defined as  $A = (L, G, l_0, L_E)$ , where  $L$  is a finite set of locations,  $G \subseteq L \times Ops \times L$ ,  $l_0 \in L$  is a unique initial location and  $L_E \subseteq L$  is a set of error locations.



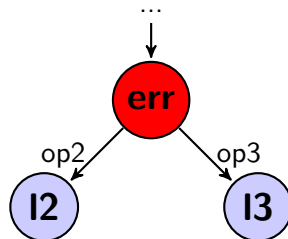


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# Error sink rule (Rule 1)

## Definition

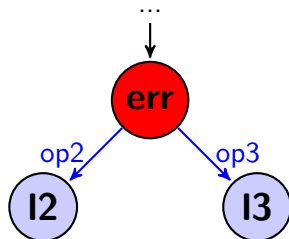
Let  $l_e \in L_E$  with  $outdegree(l_e) \geq 1$   
and  $G_{out} = \{(l_e, op_x, l_x) \in G\}$ .  
The error sink rule removes all edges  
 $g \in G_{out}$ .



# Error sink rule (Rule 1)

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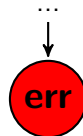
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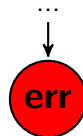
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## Complexity

$\mathcal{O}(k)$ , where  $k$  is the outdegree of  $l_e$

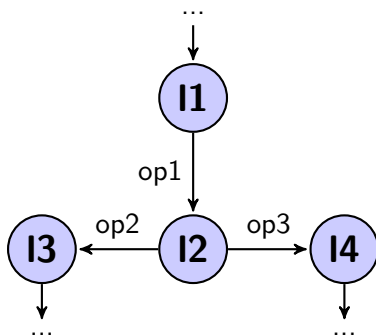
# Sequence rule (Rule 2)

## Definition

Let  $l_x \in L \setminus L_E$  with  $\text{indegree}(l_x) = 1$ ,  $\text{outdegree}(l_x) \geq 1$ ,  $l_x \notin \text{succ}(l_x)$  and the edge  $(l_y, \text{op}_y, l_x) \in G$ ,

$G_{\text{out}} = \{(l_x, \text{op}_x, l_z) \in G\}$ .

The sequence rule adds new edges with  $(l_y, \text{op}_y; \text{op}_x, l_z) \forall g \in G_{\text{out}}$  to the CFA and removes all edges  $g \in \{(l_y, \text{op}_y, l_x) \in G\} \cup G_{\text{out}}$ .



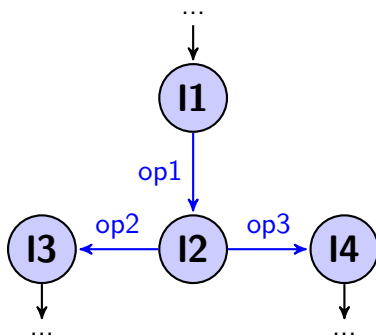
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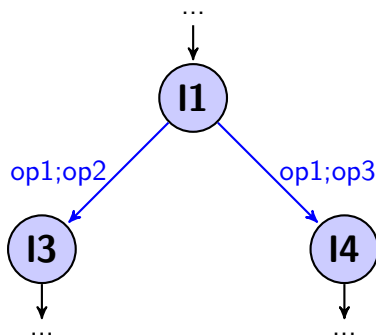
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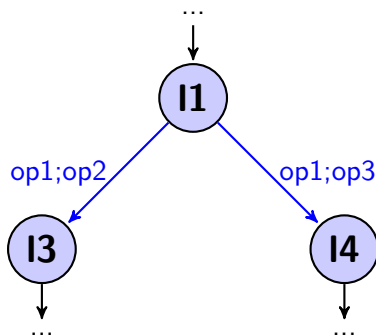
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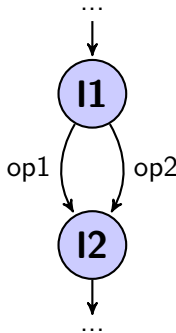
$\mathcal{O}(k)$ , where  $k$  is the outdegree of  $l_y$

# Choice rule (Rule 3)

## Definition

Let  $l_x \in L \setminus L_E$  with  $l_y \in \text{succ}(l_x)$ ,  
 $G_{con} = \text{connecting\_edges}(l_x, l_y)$  and  
 $|G_{con}| = n > 1$ .

The choice rule adds a new edge  
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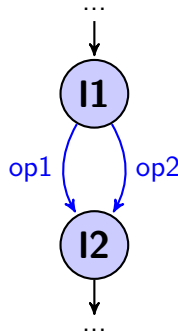


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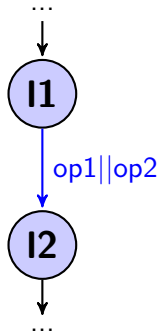


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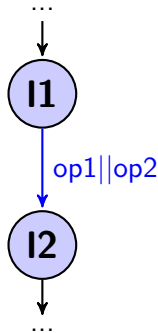


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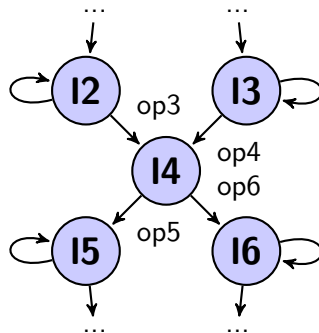
## Complexity

$\mathcal{O}(k)$ , where  $k$  is the outdegree of  $l_x$

# Advanced sequence rule (Rule 4)

## Definition

Let  $l_x \in L \setminus L_E$  with  $\text{indegree}(l_x) > 1$ ,  $\text{outdegree}(l_x) > 1$ ,  $l_x \notin \text{succ}(l_x)$  and  $G_{in} = \{(l_y, op_y, l_x) \in G\}$ ,  $G_{out} = \{(l_x, op_x, l_z) \in G\}$ .  
The sequence rule adds new edges with  $(l_y, op_y; op_x, l_z)$  for all possible connections of edges in  $G_{in}$  and  $G_{out}$  to the CFA and removes all edges  $g \in G_{in} \cup G_{out}$ .



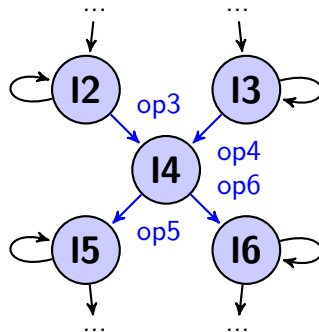
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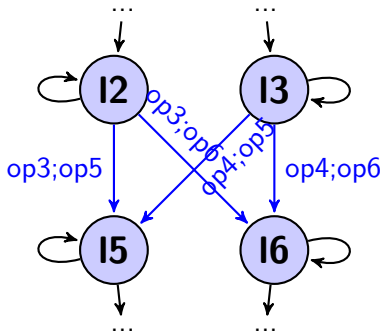
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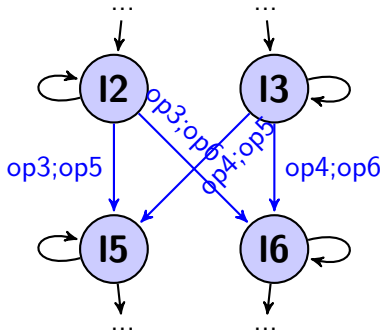
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## Complexity

$\mathcal{O}(k \cdot m)$ , where  $k$  is the outdegree of  $l_x \in L$  and  $m$  is the indegree of  $l_x \in L$

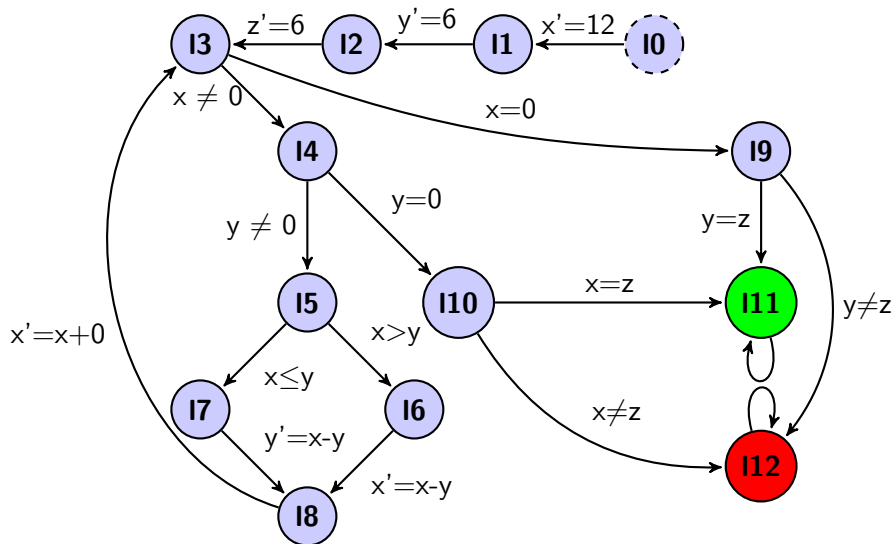
- Rule 1:  $\mathcal{O}(k)$ , where  $k$  is the outdegree of  $l_e$
- Rule 2:  $\mathcal{O}(k)$ , where  $k$  is the outdegree of  $l_y \in L$
- Rule 3:  $\mathcal{O}(k)$ , where  $k$  is the outdegree of  $l_y$
- Rule 4:  $\mathcal{O}(k \cdot m)$ , where  $k$  is the outdegree of  $l_x \in L$  and  $m$  is the indegree of  $l_x \in L$

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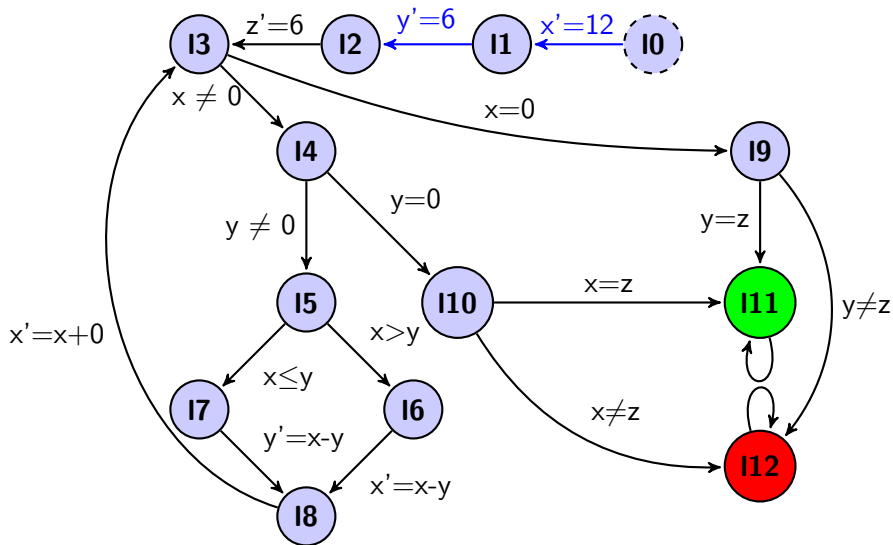
## Complexity in total

$$\mathcal{O}(|G| \cdot |L| \cdot k \cdot m)$$

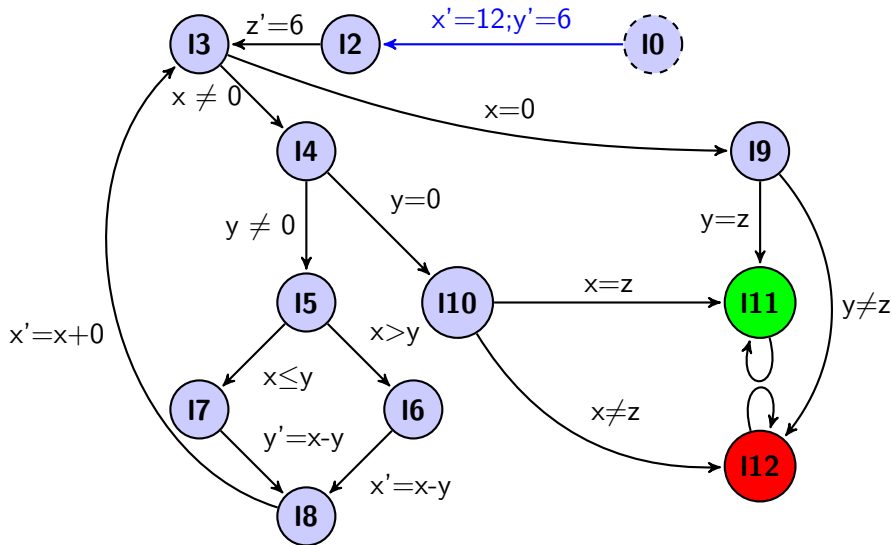
# Example



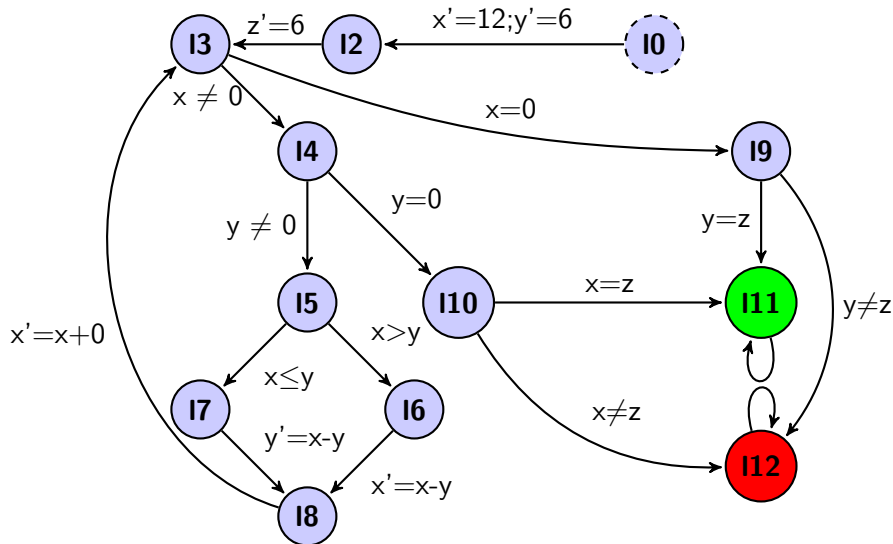
# Example



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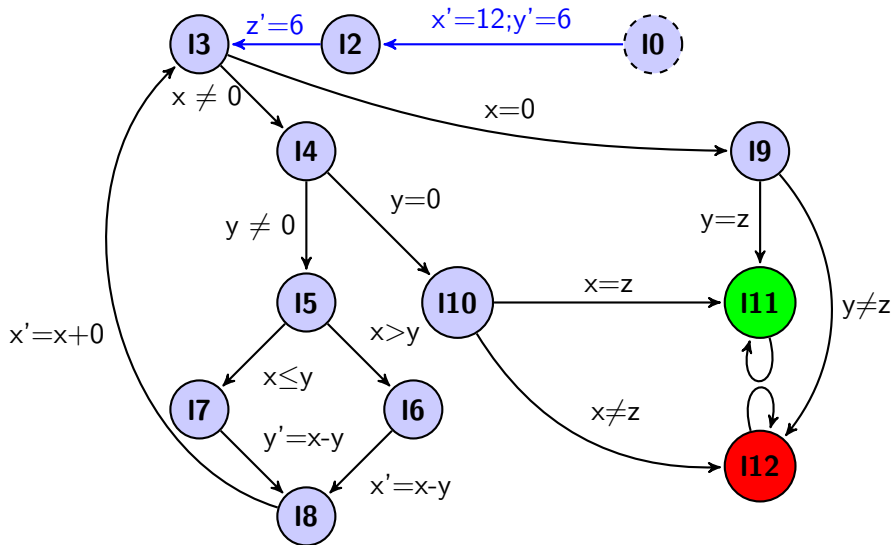


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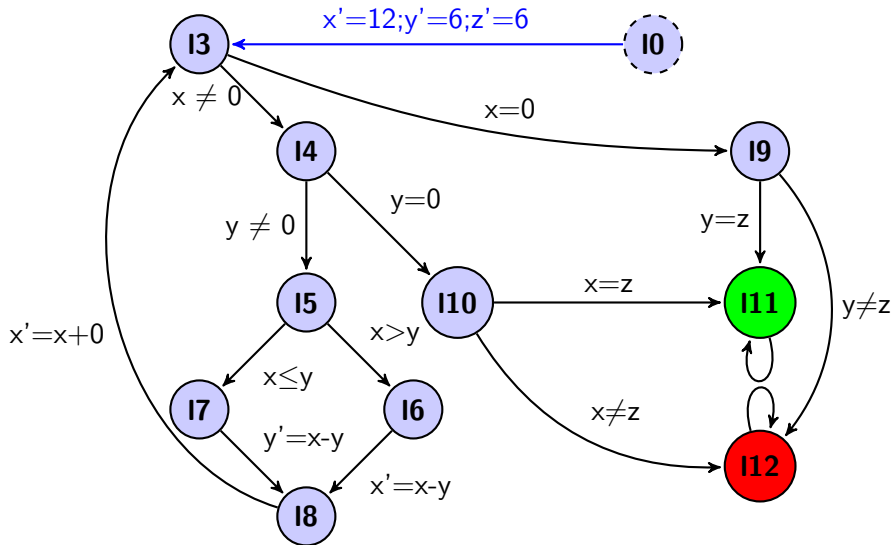




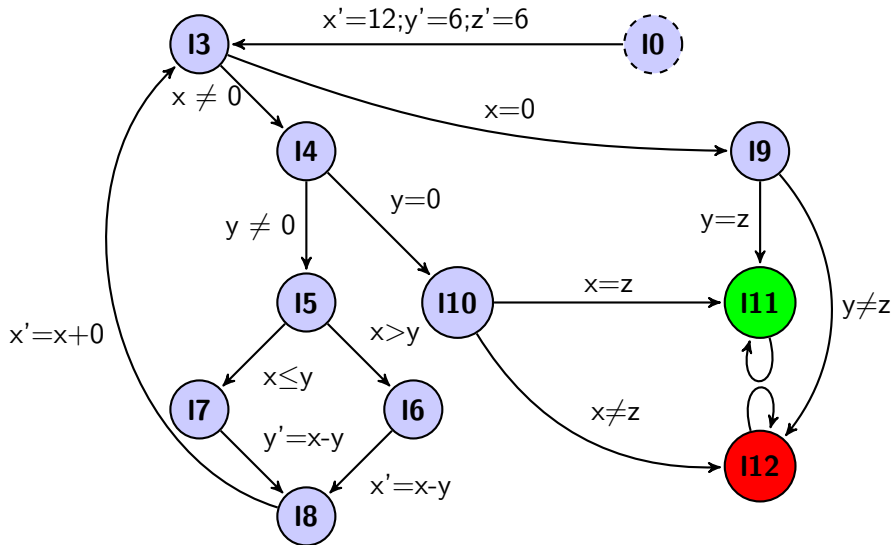
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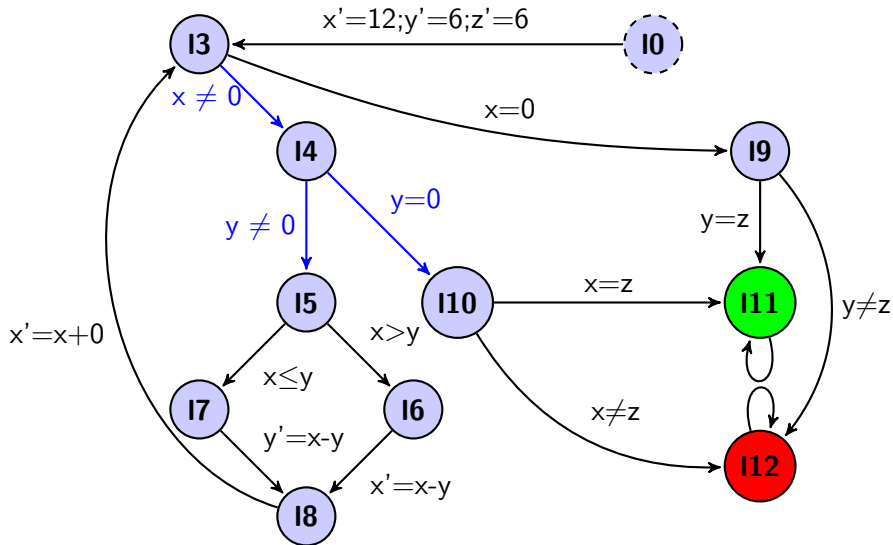
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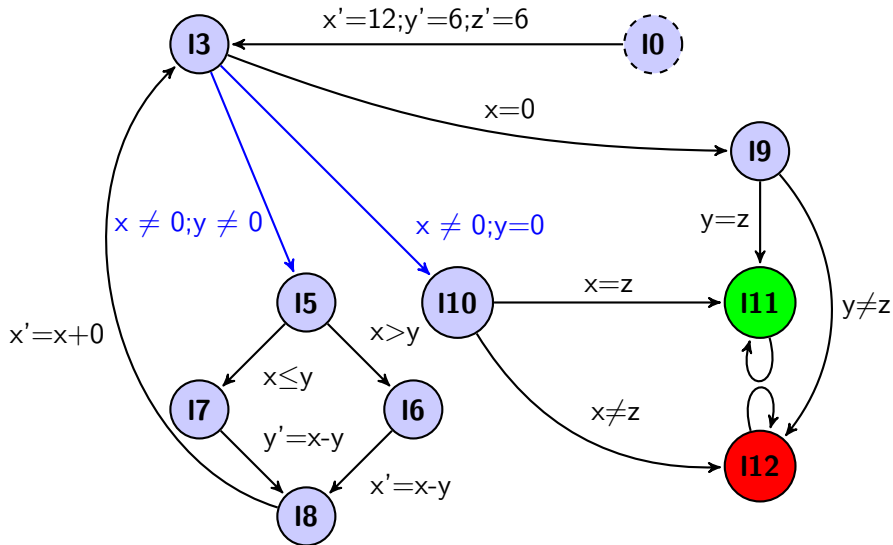
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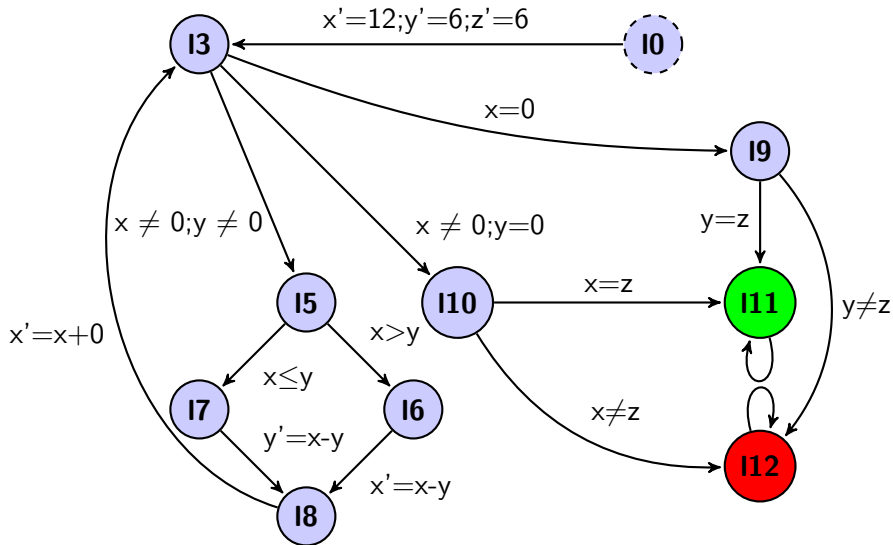
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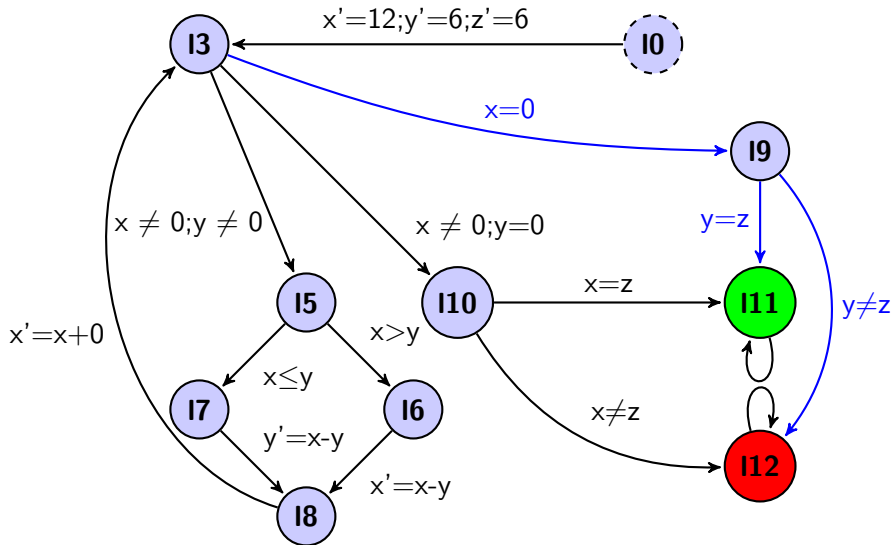
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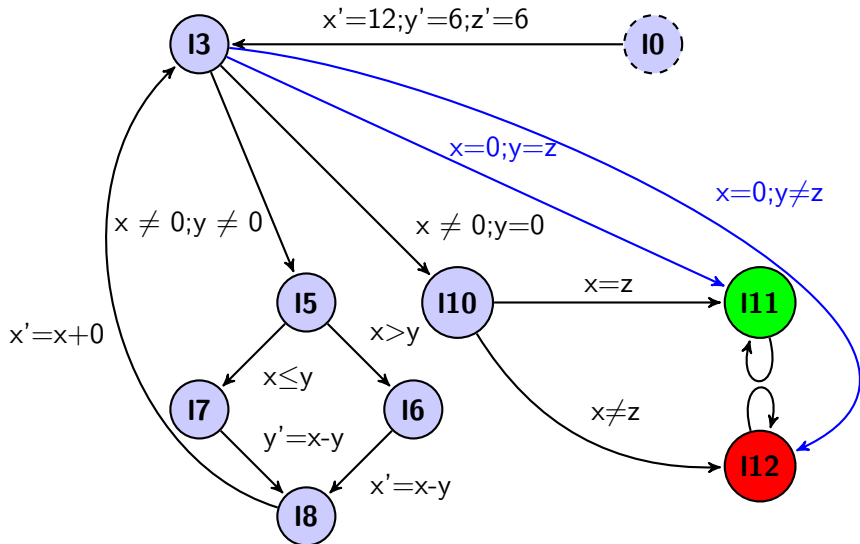
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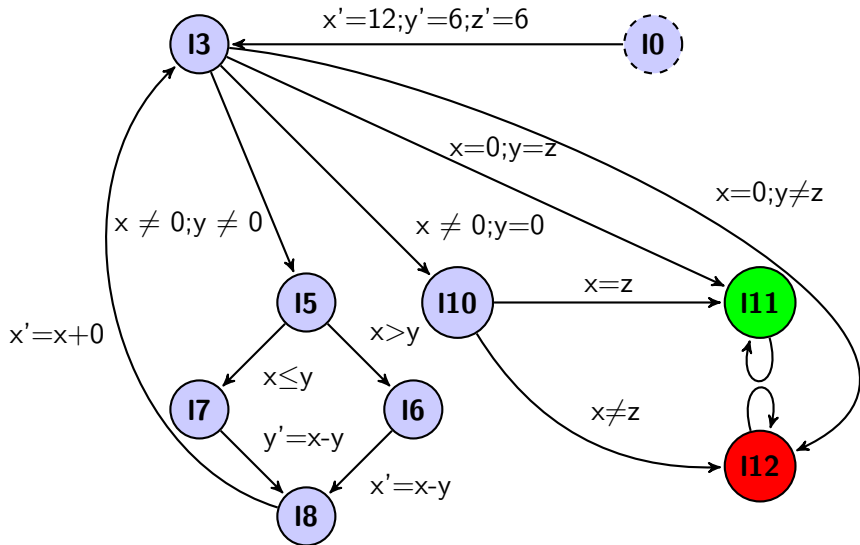


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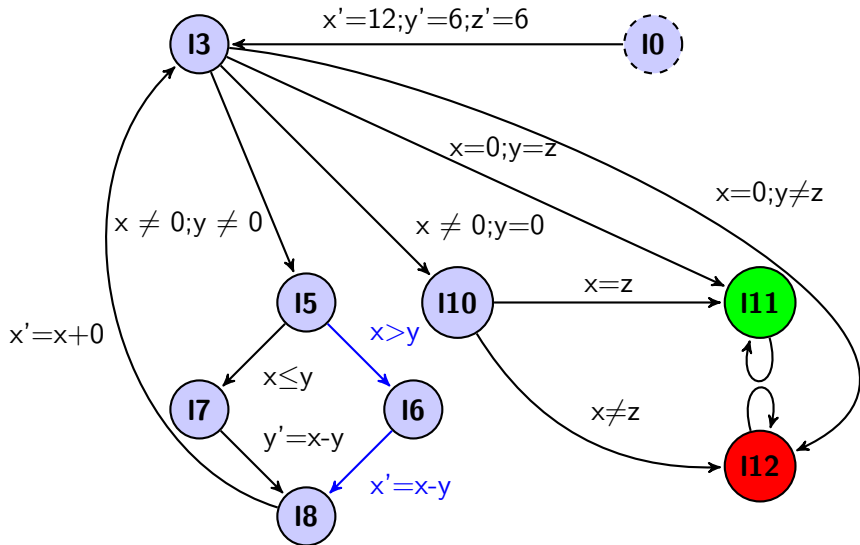




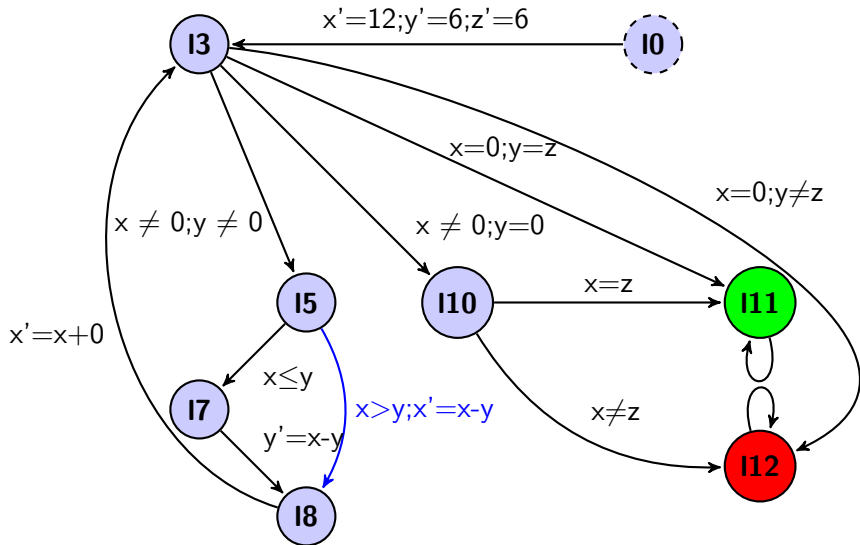
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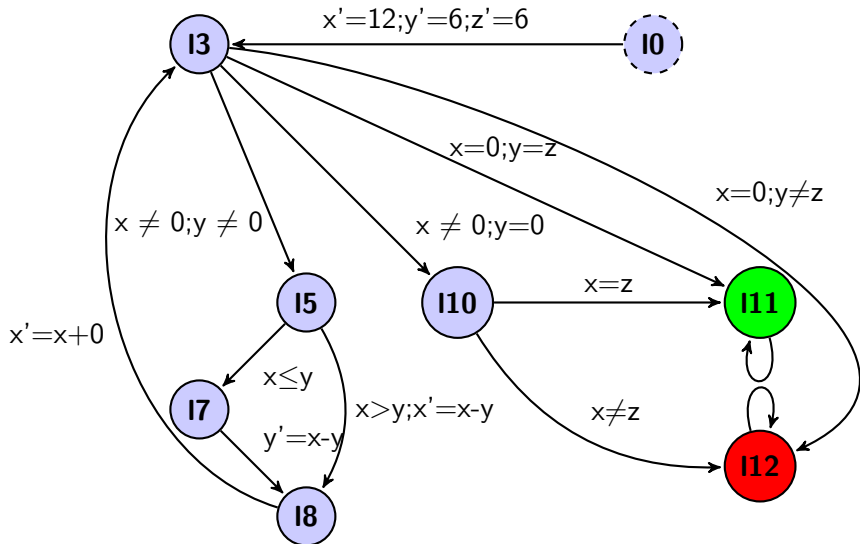
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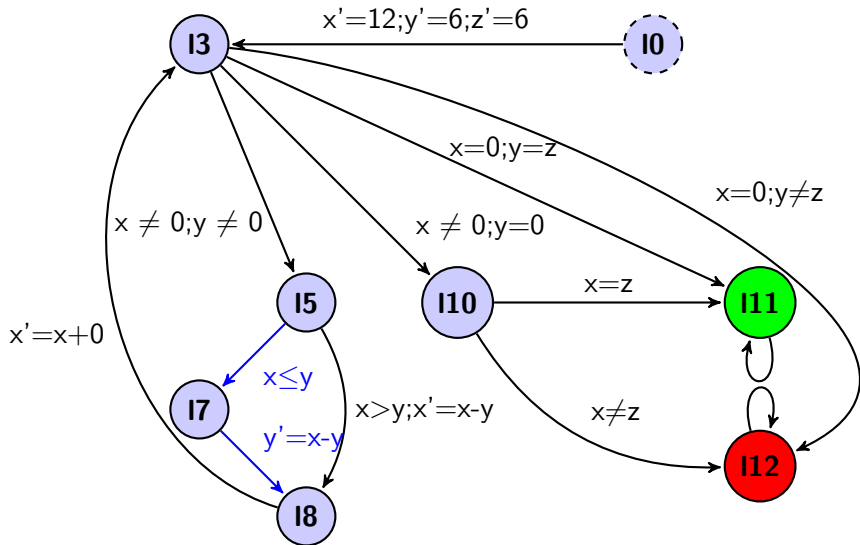
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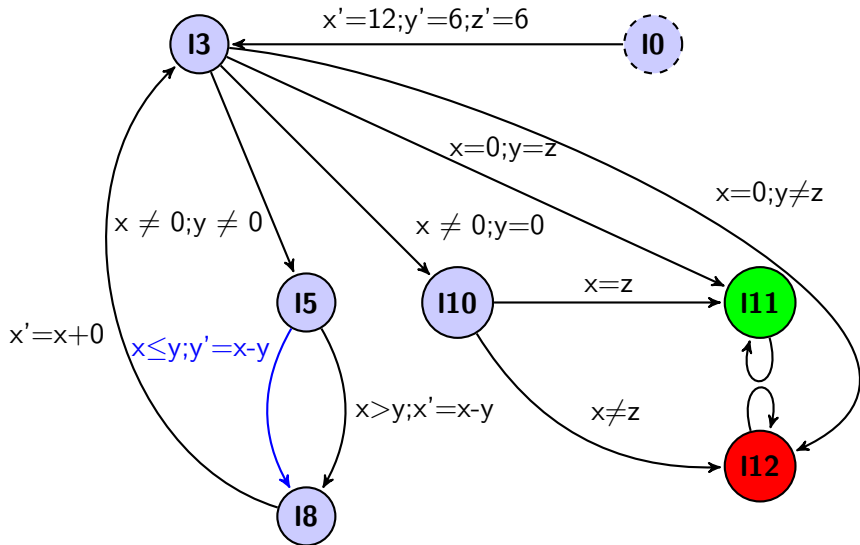
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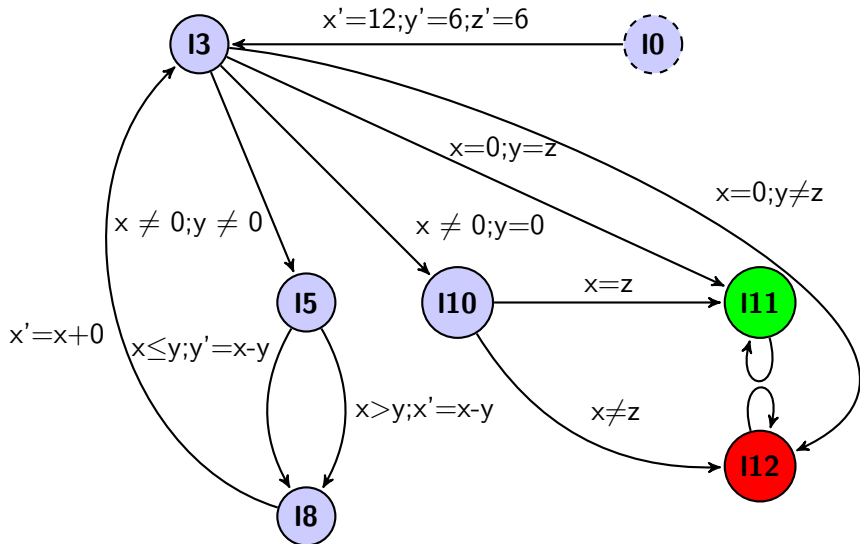
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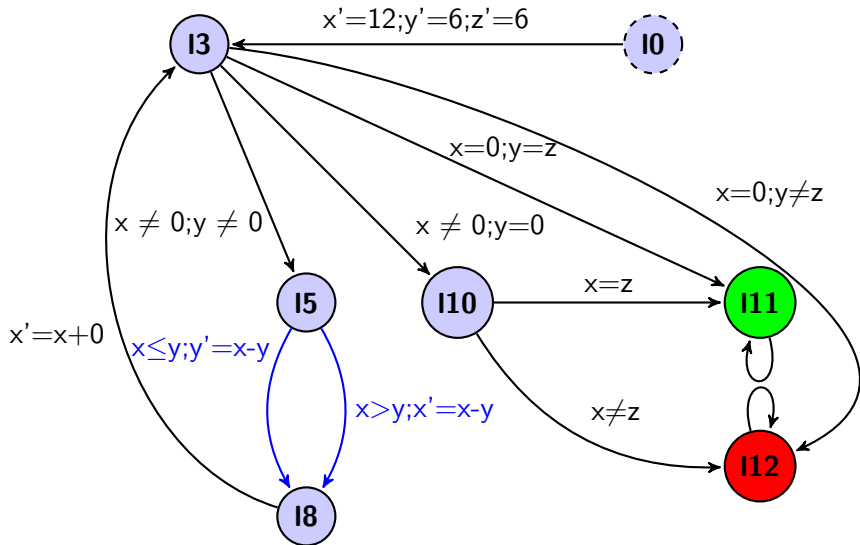
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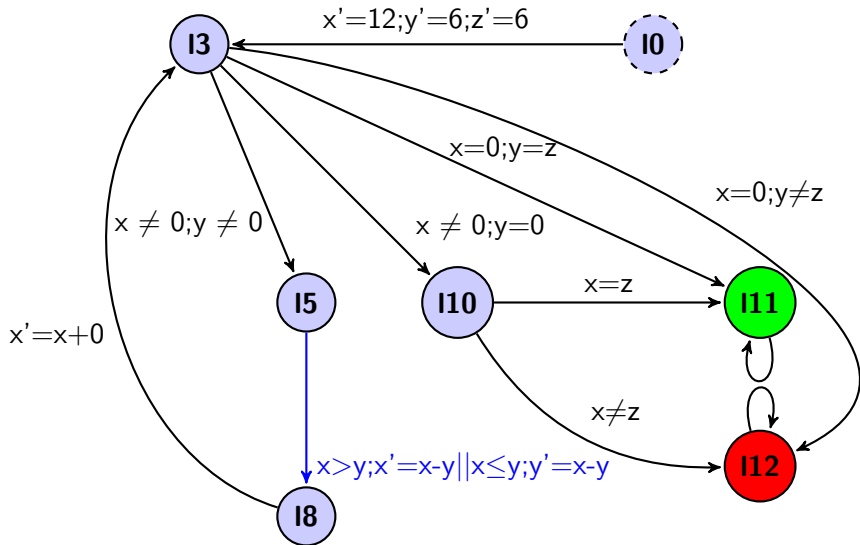


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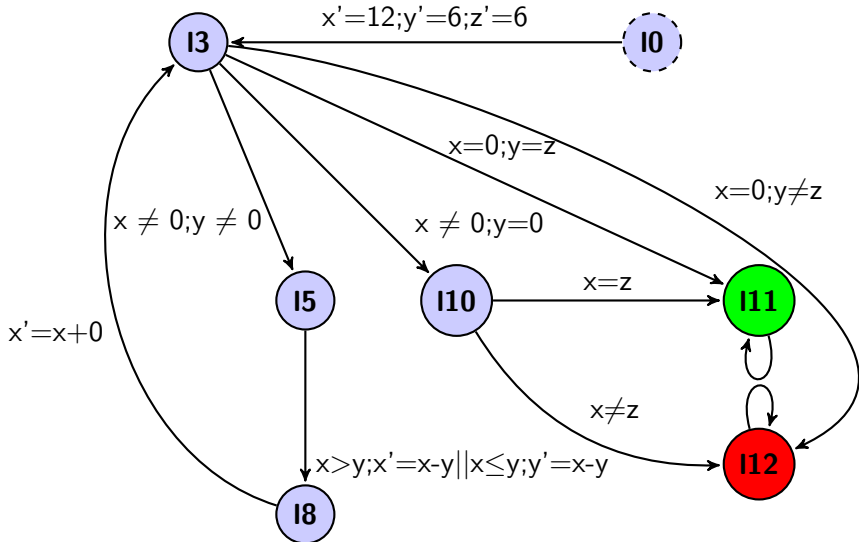




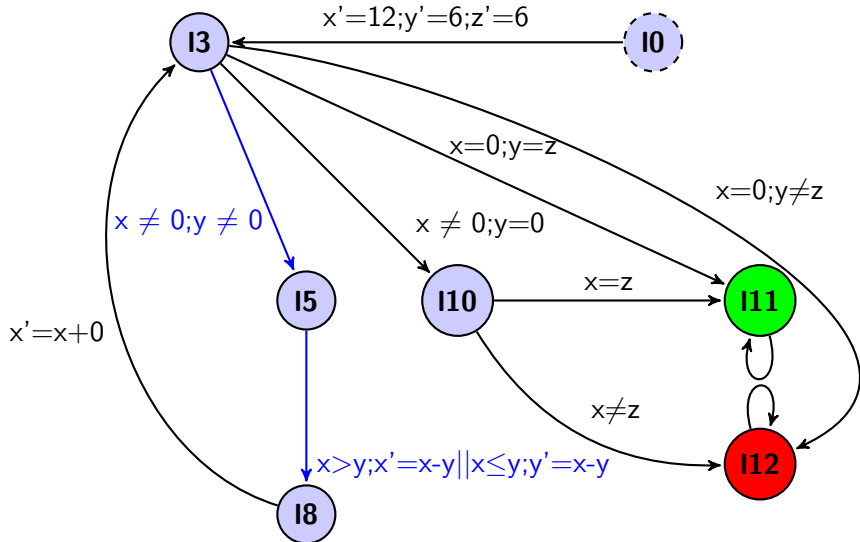
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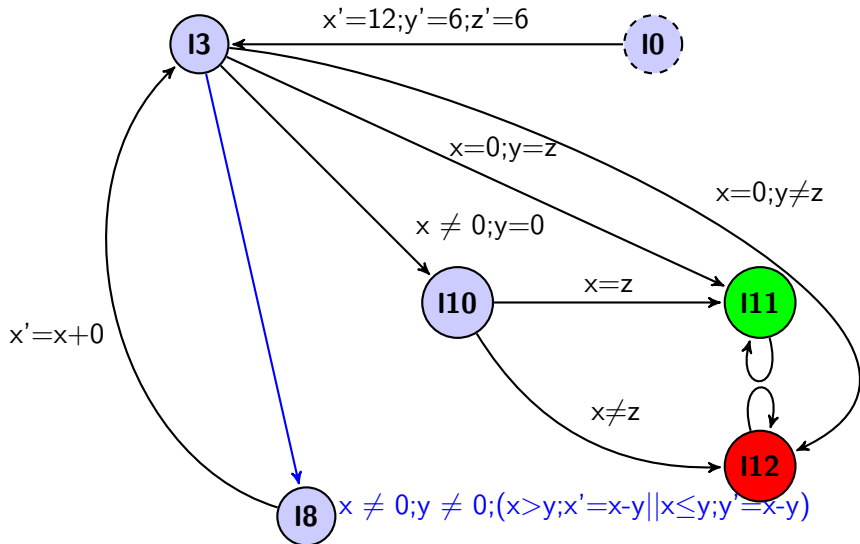
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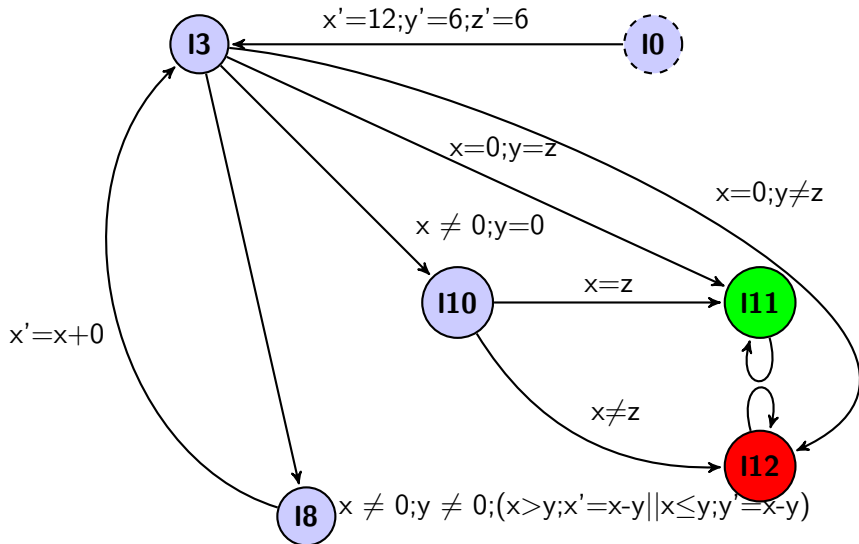
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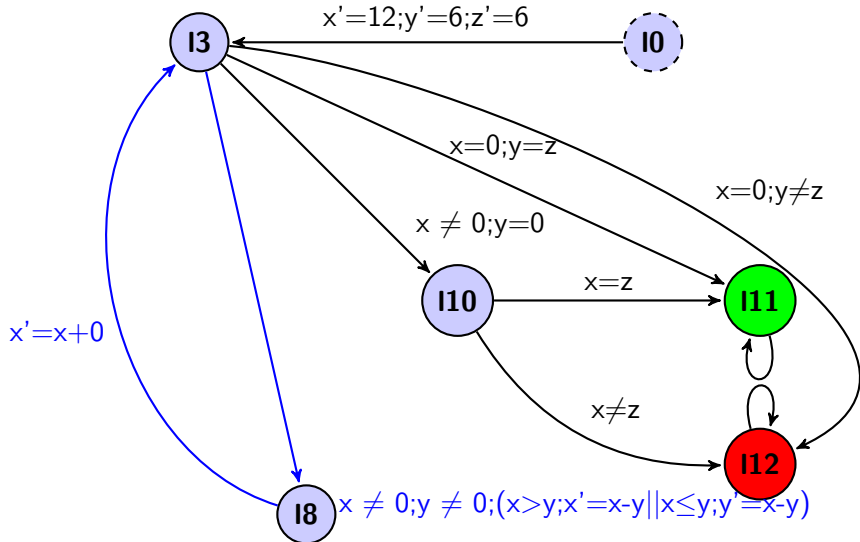
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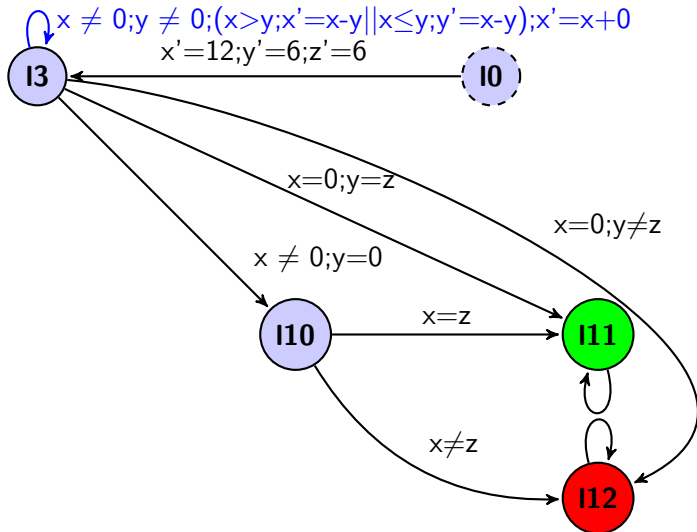
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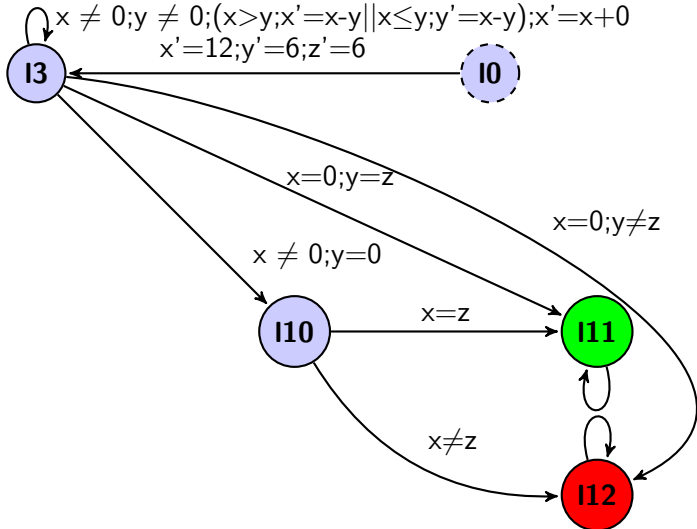
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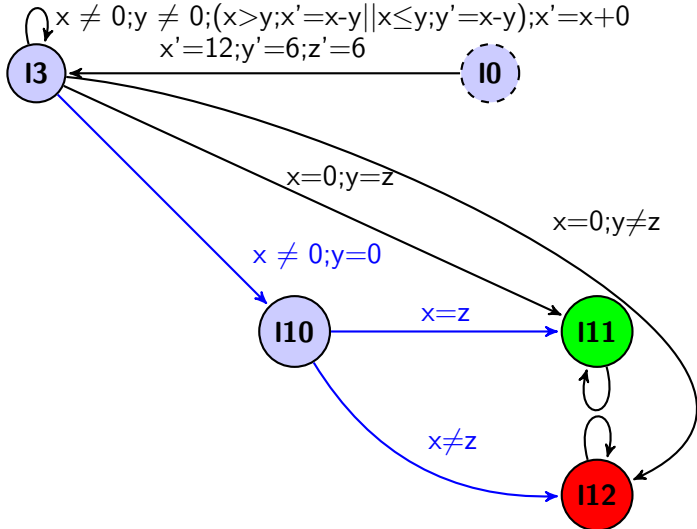


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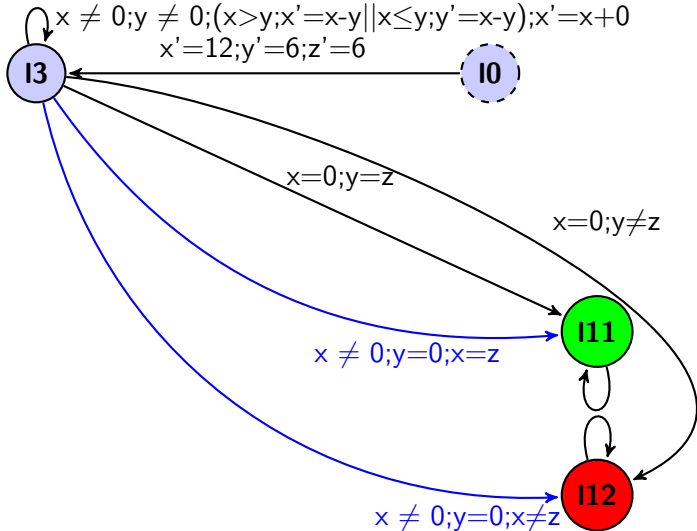




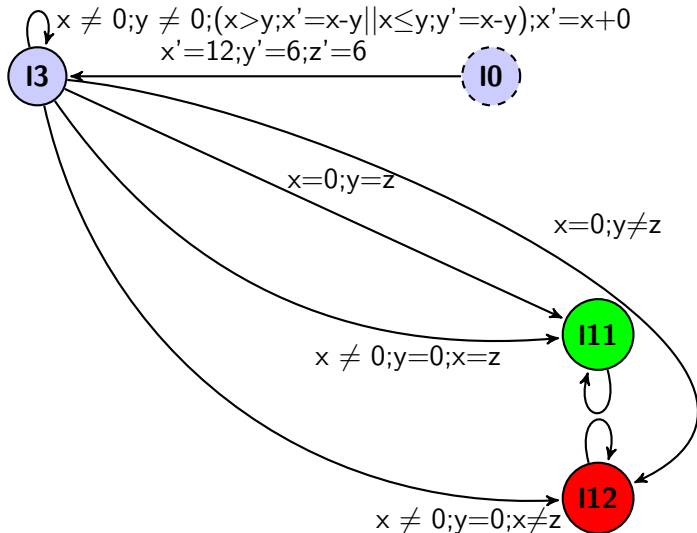
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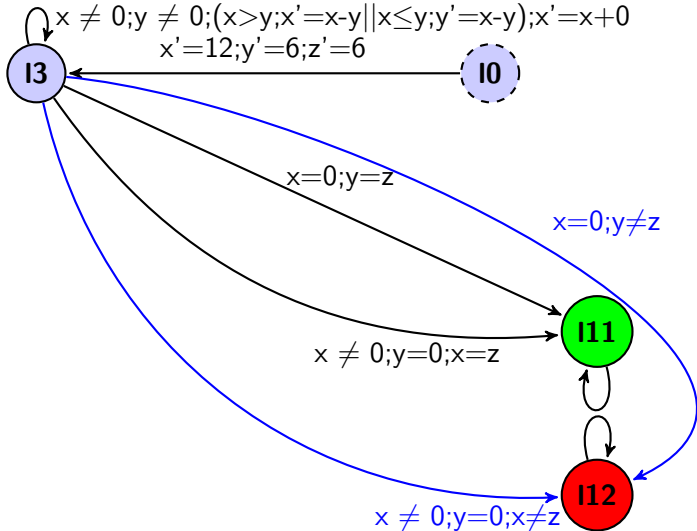
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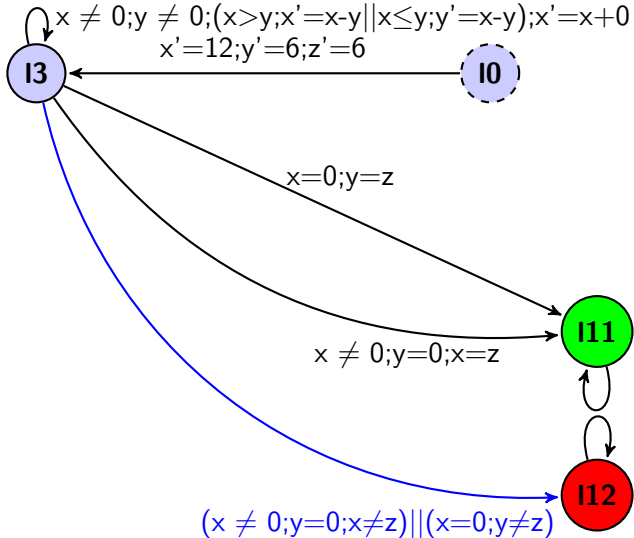
# Example



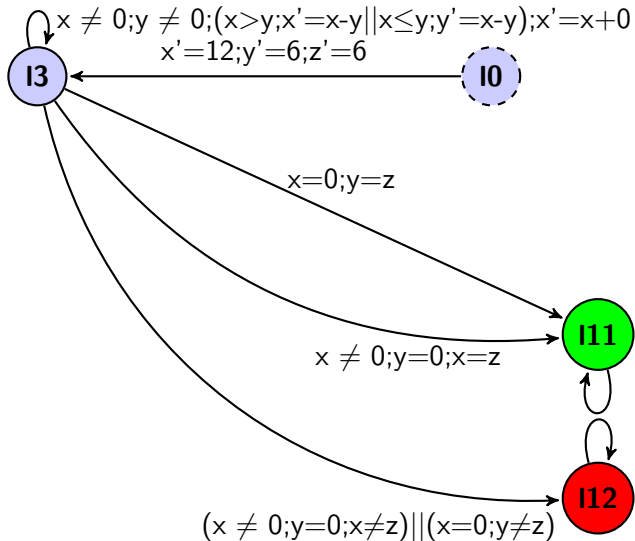
# Example



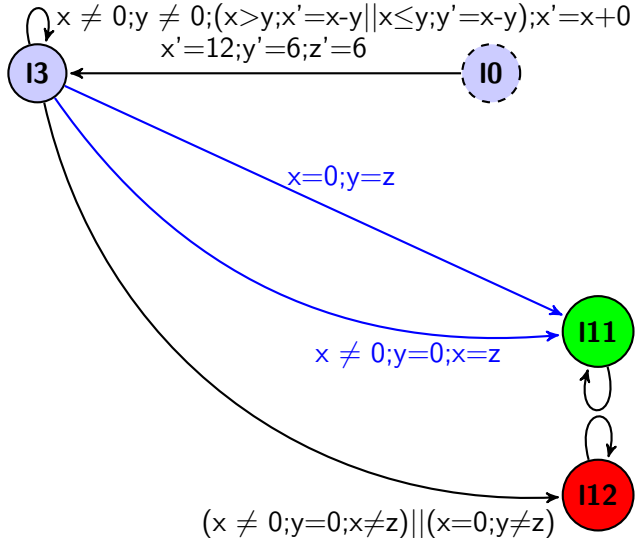
# Example



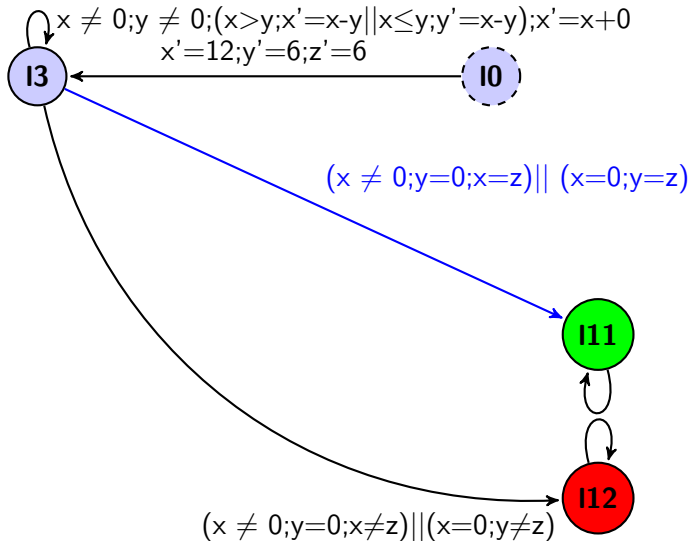
# Example



# Example

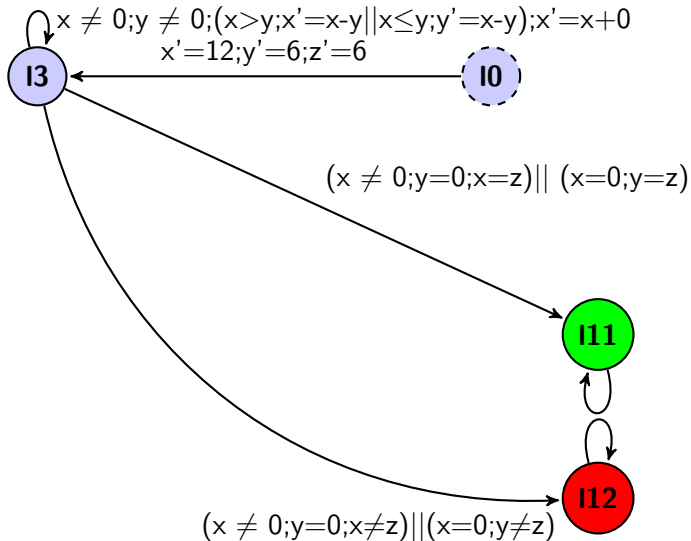


# Example

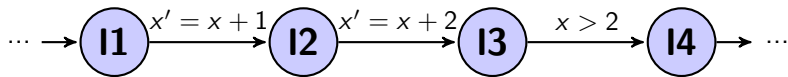




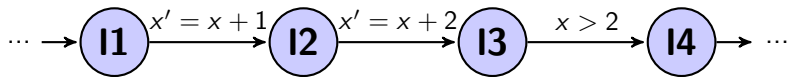
# Example



# Variable Substitution

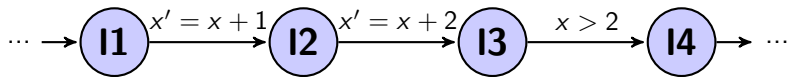


# Variable Substitution

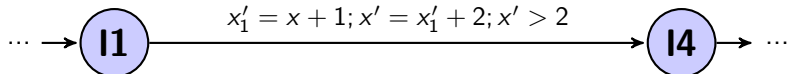


Model Checker interprets an assignment as  $f : VAR \rightarrow VAR'$ .

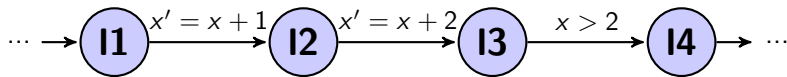
# Variable Substitution



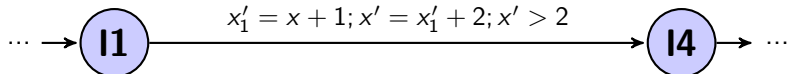
Model Checker interprets an assignment as  $f : VAR \rightarrow VAR'$ .



# Variable Substitution

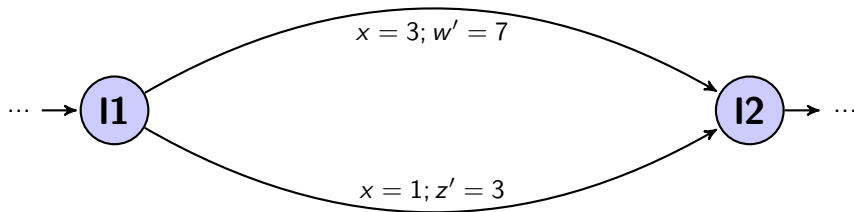


Model Checker interprets an assignment as  $f : VAR \rightarrow VAR'$ .

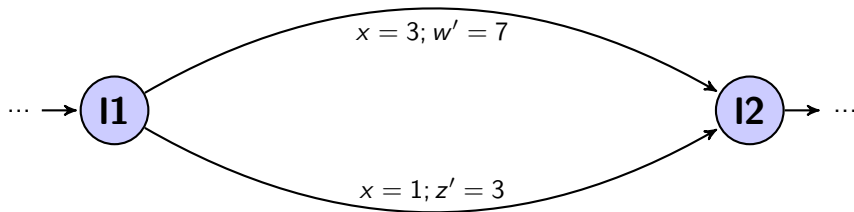


A sequential execution of assignments looks as follow:  $f_2(f_1(x))$

# Variable Propagation

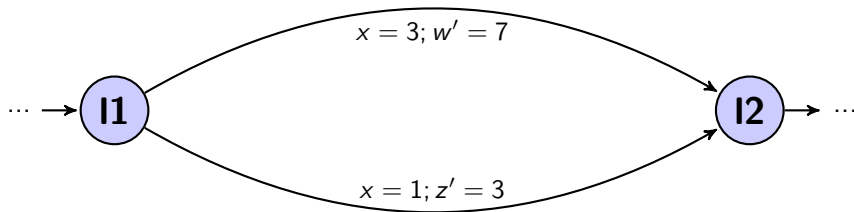


# Variable Propagation

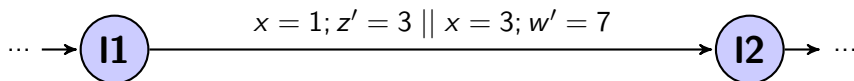


Ensure that model checker does not skip a variable assignment

# Variable Propagation

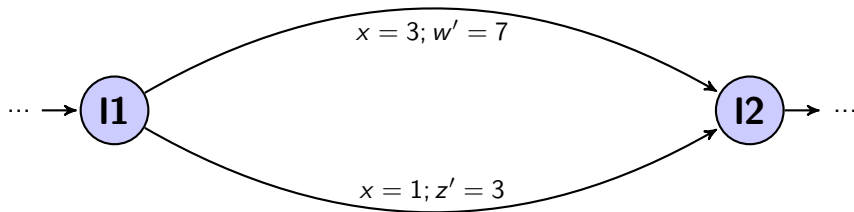


Ensure that model checker does not skip a variable assignment

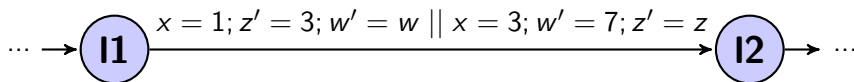




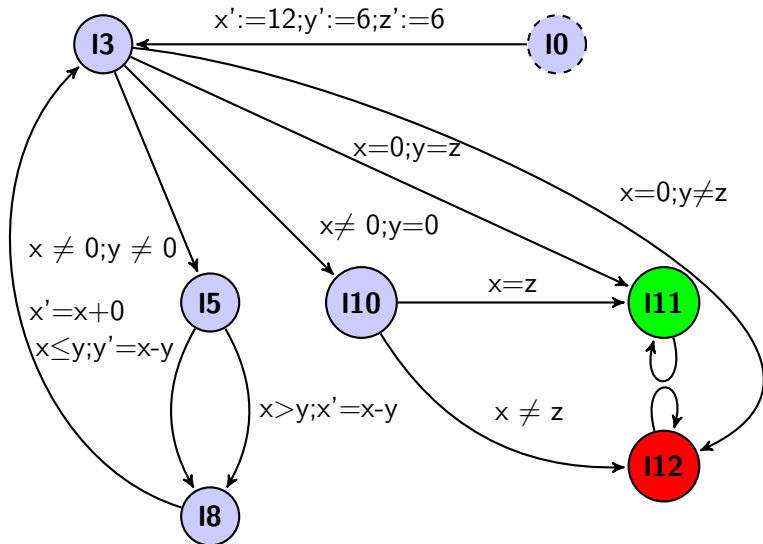
# Variable Propagation



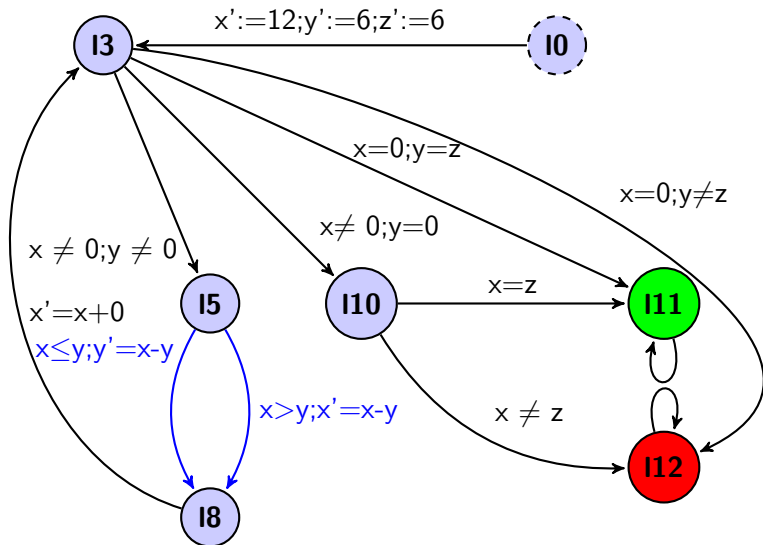
Ensure that model checker does not skip a variable assignment



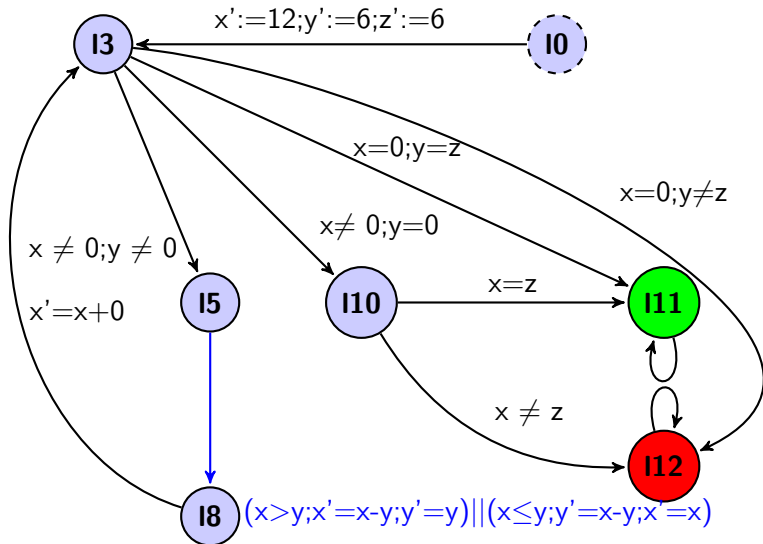
# Example



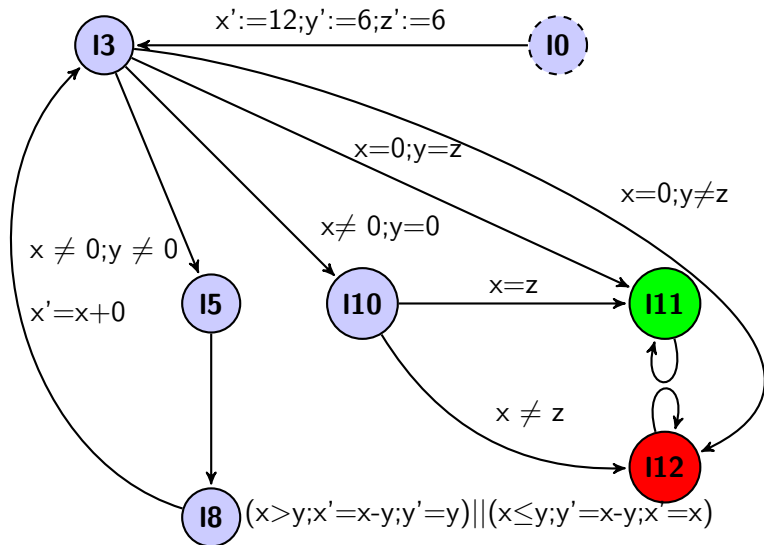
# Example



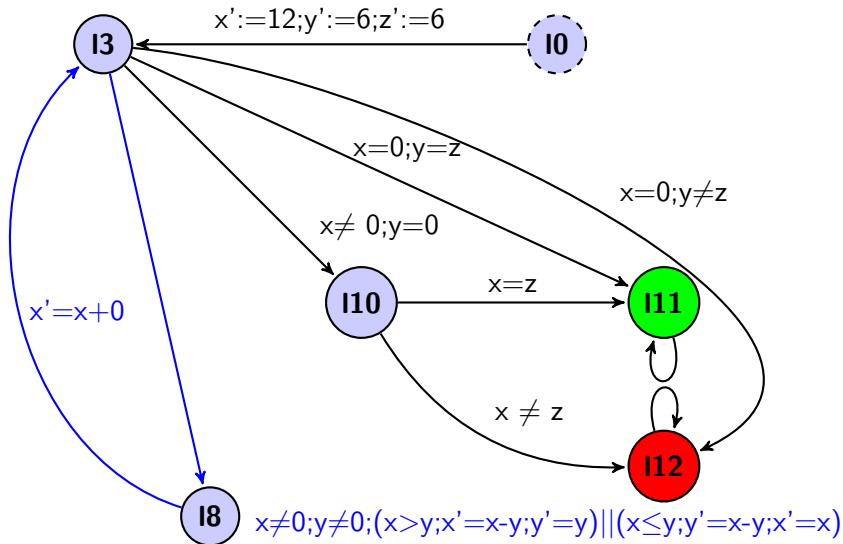
# Example



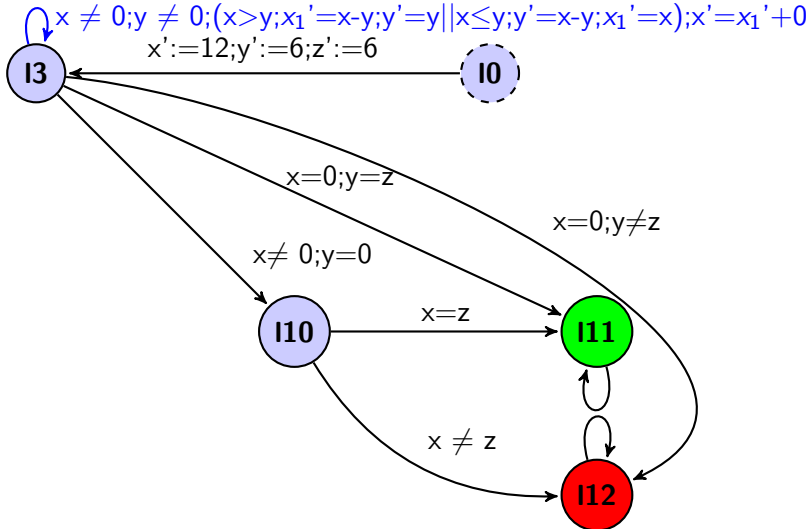
# Example



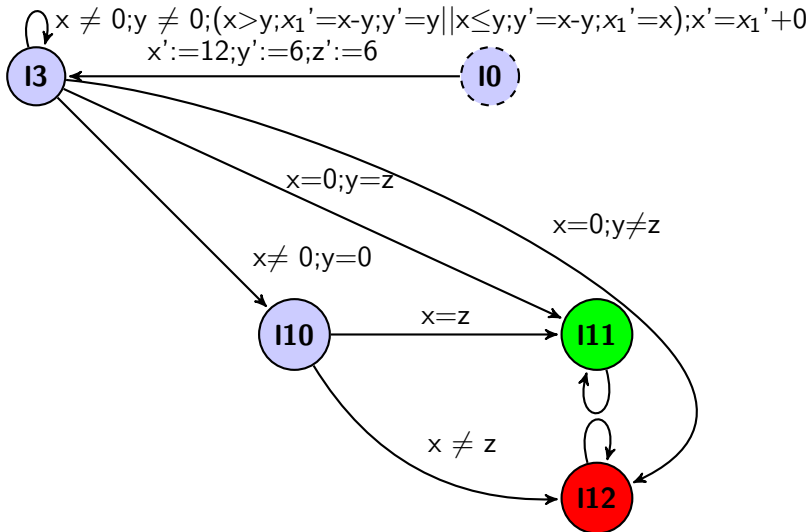
# Example



# Example



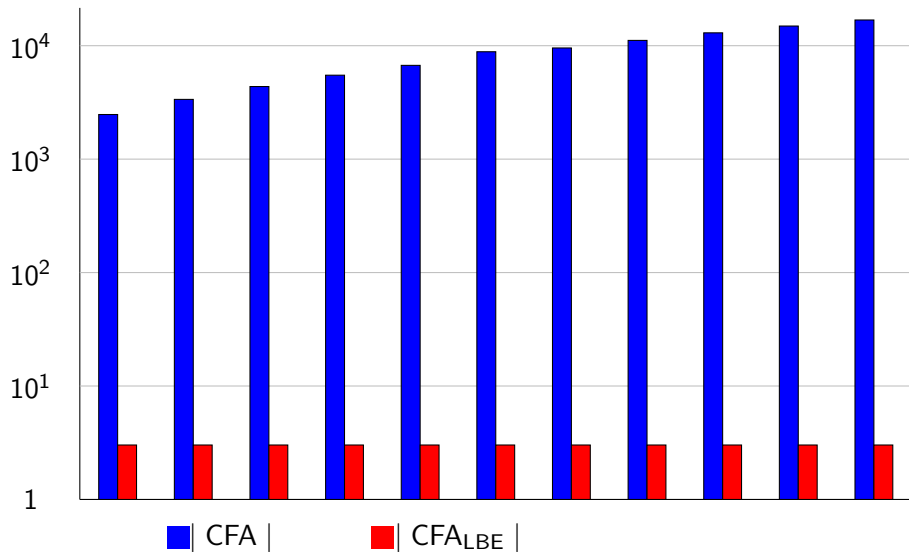
# Example



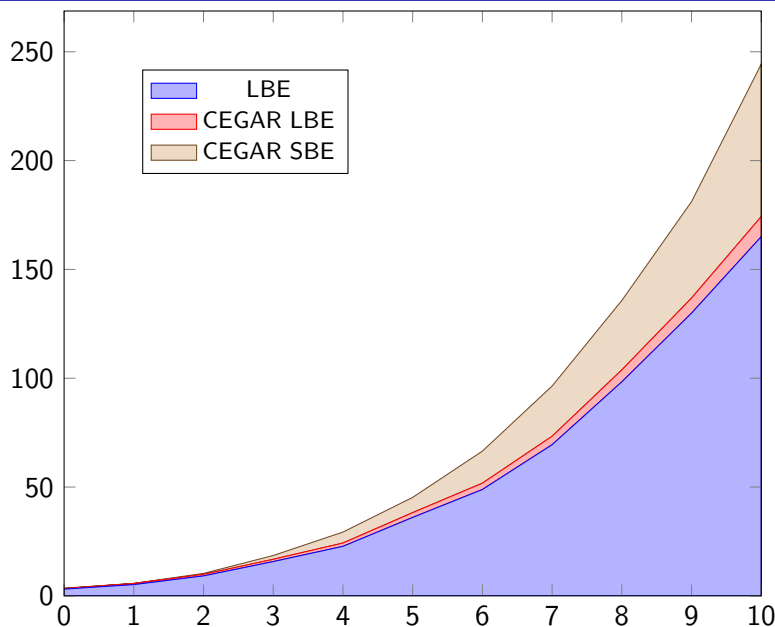


- 1 Motivation
- 2 Background
  - Program and Control-Flow Automaton
  - Model Checker
- 3 Large-Block Encoding
  - Summarization
  - Post-processing
- 4 Evaluation
- 5 Conclusion

# CFA Reduction



# Runtime Analysis (CEGAR)



- 1 Motivation
- 2 Background
  - Program and Control-Flow Automaton
  - Model Checker
- 3 Large-Block Encoding
  - Summarization
  - Post-processing
- 4 Evaluation
- 5 Conclusion



Questions?