Introduction to IC3 & IC3CFA

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Outline

Inductive Invariants

Finite State Inductive Strengthening

IC3

IC3 on Control Flow Automata

Inductive Invariants

Inductivity

A property P is inductive if it satisfies initiation and consecution:

$$\text{Initiation: } I \Rightarrow P$$

$$\text{Consecution: } P \wedge T \Rightarrow P'$$

Inductive invariant [MP95]

Given a property P of a system, if P is inductive on S it is an invariant on S.

[MP95] Zohar Manna and Amir Pnueli. *Temporal verification of reactive systems - safety*. Springer, 1995

Inductive Invariants

Inductivity

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Inductive invariant [MP95]

Given a property P of a system, if P is inductive on S it is an invariant on S.

Not vice versa

Even if P is an invariant on S, it may not be inductive.

[MP95] Zohar Manna and Amir Pnueli. *Temporal verification of reactive systems - safety*. Springer, 1995

Finite State Inductive Strengthening

Proving a property P on a system S

Proving that P holds on S in general is hard. But if P would be inductive, it would be trivial. If we only had a way to make P inductive ...

Inductive Strengthening [BM07]

Try to find a formula F that is an inductive strengthening of P, i.e.

$$\text{Initiation: } I \Rightarrow P \wedge F$$

$$\text{Consecution: } P \wedge F \wedge T \Rightarrow P' \wedge F'$$

[BM07] Aaron R. Bradley and Zohar Manna. "Checking Safety by Inductive Generalization of Counterexamples to Induction". In: *FMCAD*. 2007, pp. 173–180

Finite State Inductive Strengthening

The bad guys

An F-state that has a transition to a $\neg F$-state is called *Counterexample to Induction* (CTI) and is a direct wittness for why F is not inductive.

Finding CTIs

A CTI can be found using a simple satisfiability query

$$sat(F \wedge T \wedge \neg F).$$

If the query is sat, there exists a CTI and we can extract the state from the model of the solver (satisfying variable assignment).

Finite State Inductive Strengthening

Handling CTIs

After finding a CTI s we check whether there exists a *minimal inductive subclause* for $\neg s$. If not, we update P that we have to prove that s is not reachable. Otherwise we can add $\neg s$ to the strengthening F.

Finite State Inductive Strengthening

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Happy End?

FSIS terminates if either

- $P \wedge F$ becomes inductive, or
- $I \not\Rightarrow P$ anymore.

IC3

Finding F is hard

In many cases it can be pretty hard to come up with such a strengthening. Especially finding a minimal inductive subclause e that is inductive relative to $P \wedge F$ is hard.

Solution [Bra11]

Instead of computing F directly, compute a sequence $F_0, \dots, F_k$, called frames, to a find an inductive strengthening within these F_i.

[Bra11] Aaron R. Bradley. "SAT-Based Model Checking without Unrolling". In: *VMCAI*. 2011, pp. 70–87

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Frames

For a frame sequence $F_0, \ldots, F_k$ to be an inductive strengthening, the following must hold:

$$I \Rightarrow F_0 \tag{1}$$
$$F_i \Rightarrow F_{i+1} \tag{2}$$
$$F_i \Rightarrow P \tag{3}$$
$$F_i \wedge T \Rightarrow F'_{i+1} \tag{4}$$

Algorithm 1 Outer loop

function bool prove
    Check 0- and 1-step counterexamples
    Initialise frames $F_0 = I, F_1 = P$
    for k = 1 to ... **do**
        Blocking phase
        Propagation phase
        Check termination

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Handling CTIs

Given a CTI c in the last frame F_k, we check whether c is reachable from F_{k-1} in one step.

Thinking inductive:

From an F_{k-1} state that is not c, do we stay in not c after one step? In other words: Is $\neg c$ inductive relative to F_k:

$$F_k \wedge \neg c \wedge T \Rightarrow \neg c$$

How to check validity

Validity of the implication can be solved as $unsat(F_k \wedge \neg c \wedge T \wedge c)$.

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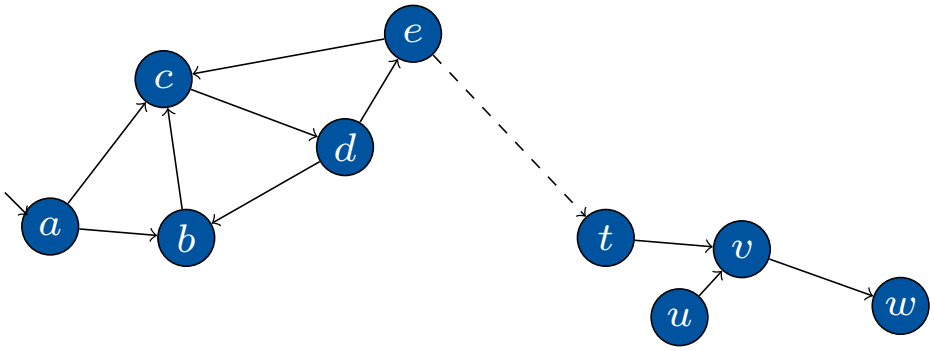
Termination

If for any frame F_i, $0 \leq i < k$ it holds that $F_i = F_{i+1}$ then

$$\begin{aligned} & F_i \wedge T \Rightarrow F_{i+1} \\ \Leftrightarrow\ & F_i \wedge T \Rightarrow F_i \end{aligned}$$

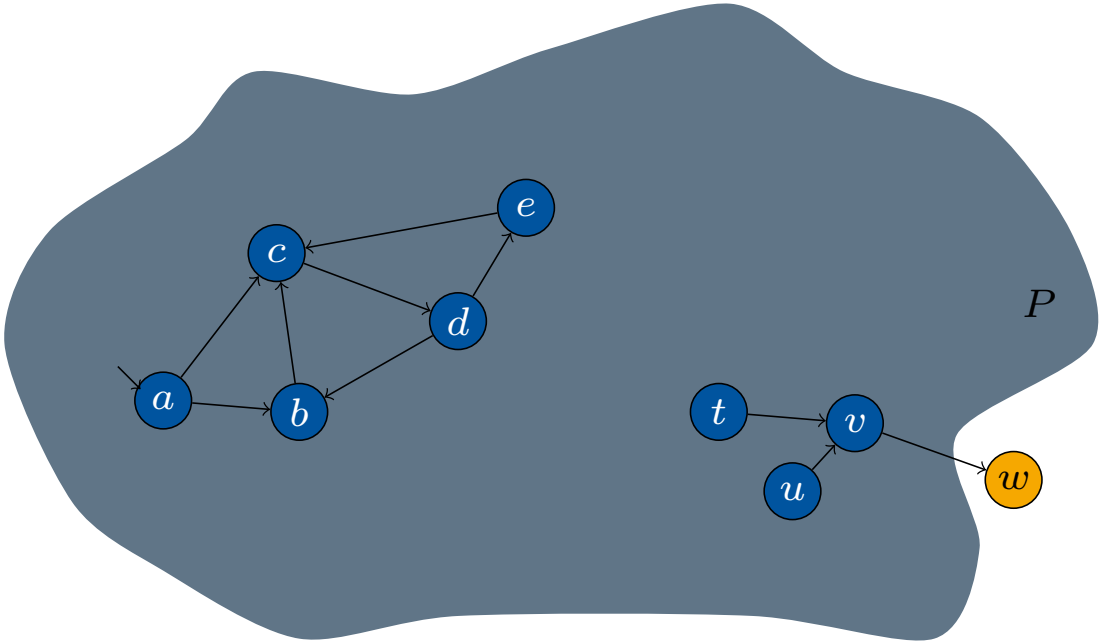
IC3

Consider the transition system $\mathcal{M} = (X, I, T)$



IC3

Consider the transition system $\mathcal{M} = (X, I, T)$ and the property $P(X)$.



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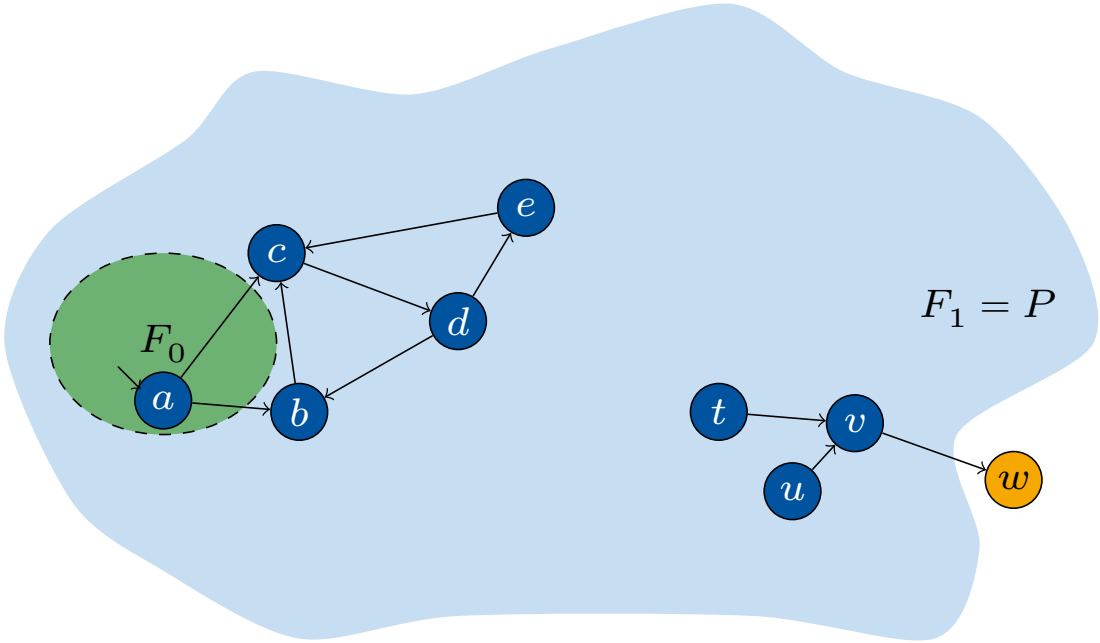

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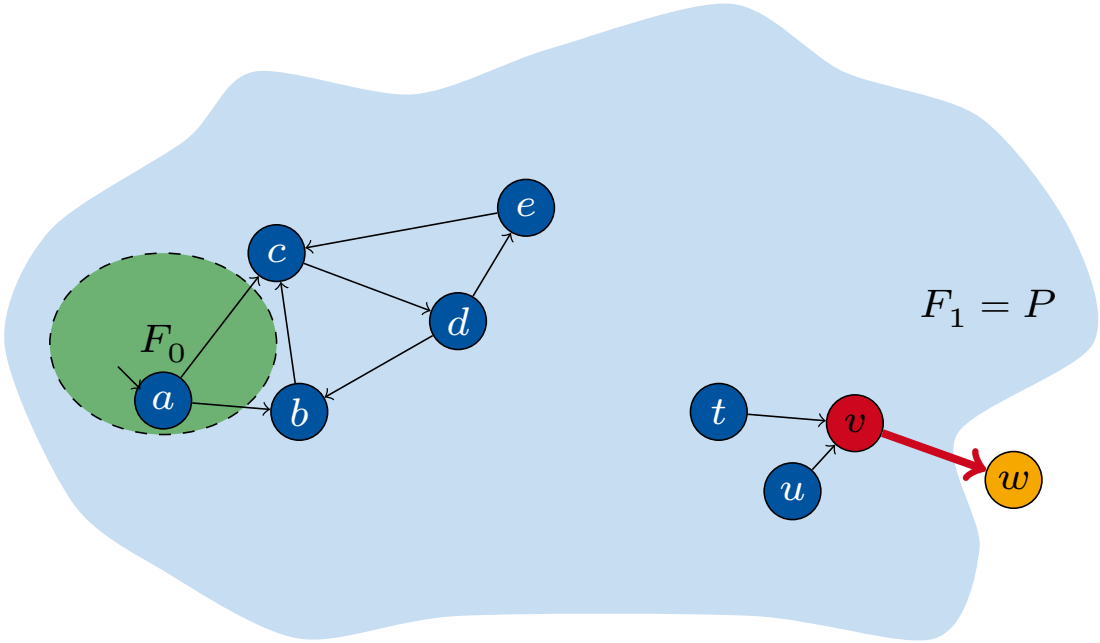

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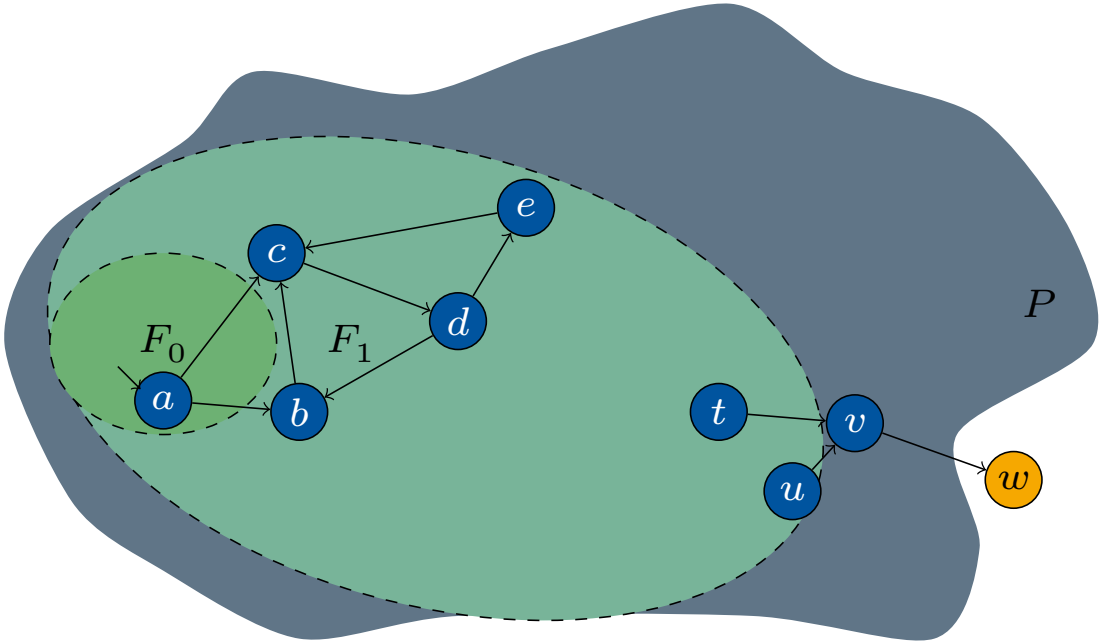

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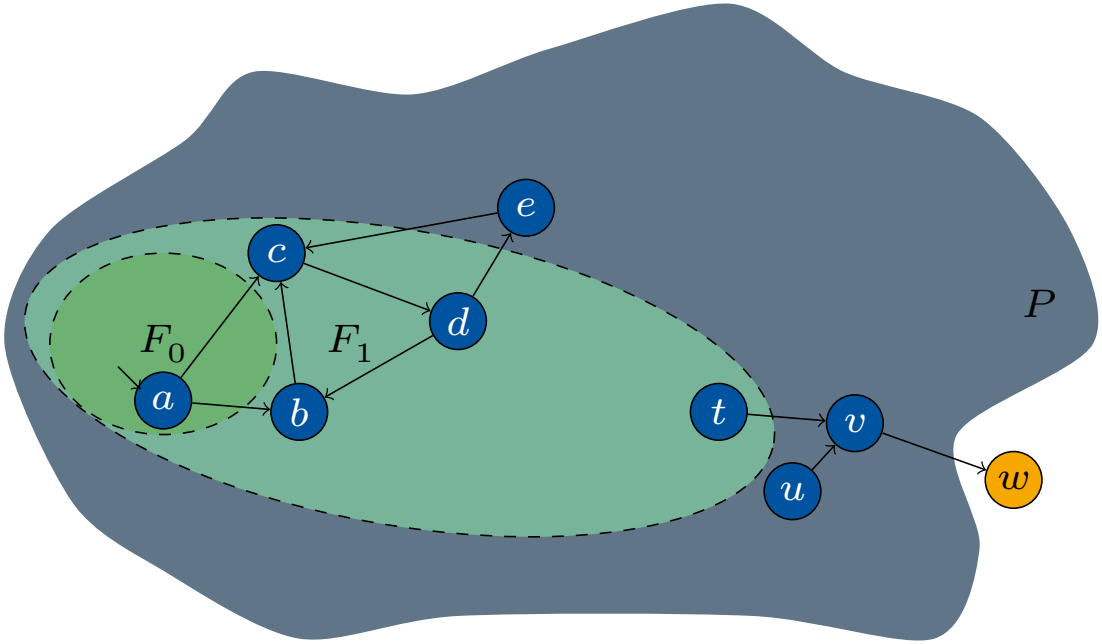

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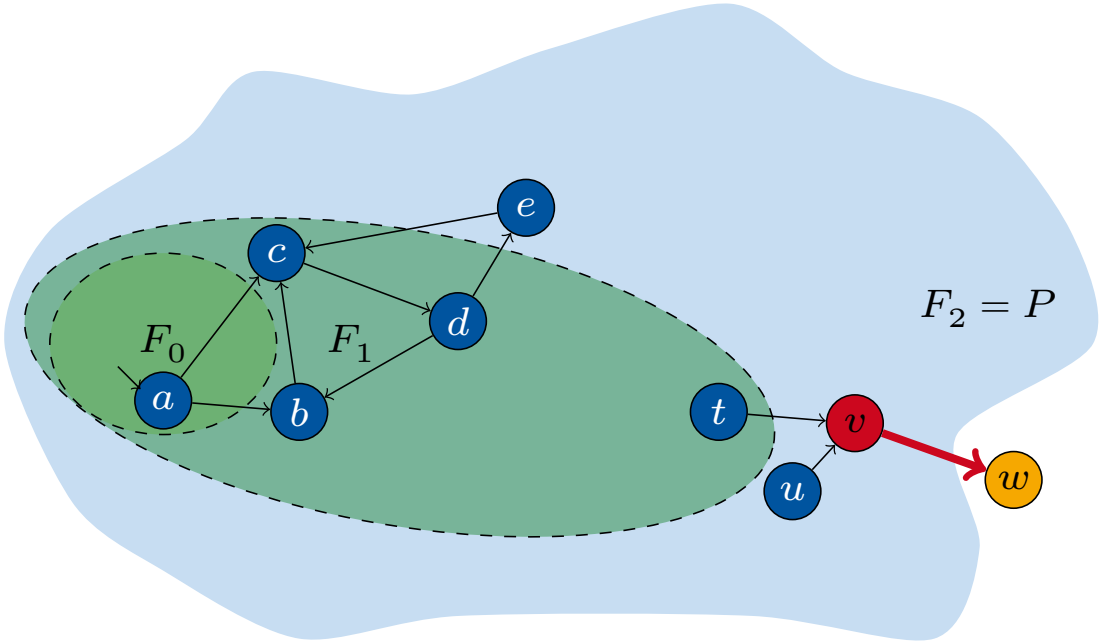

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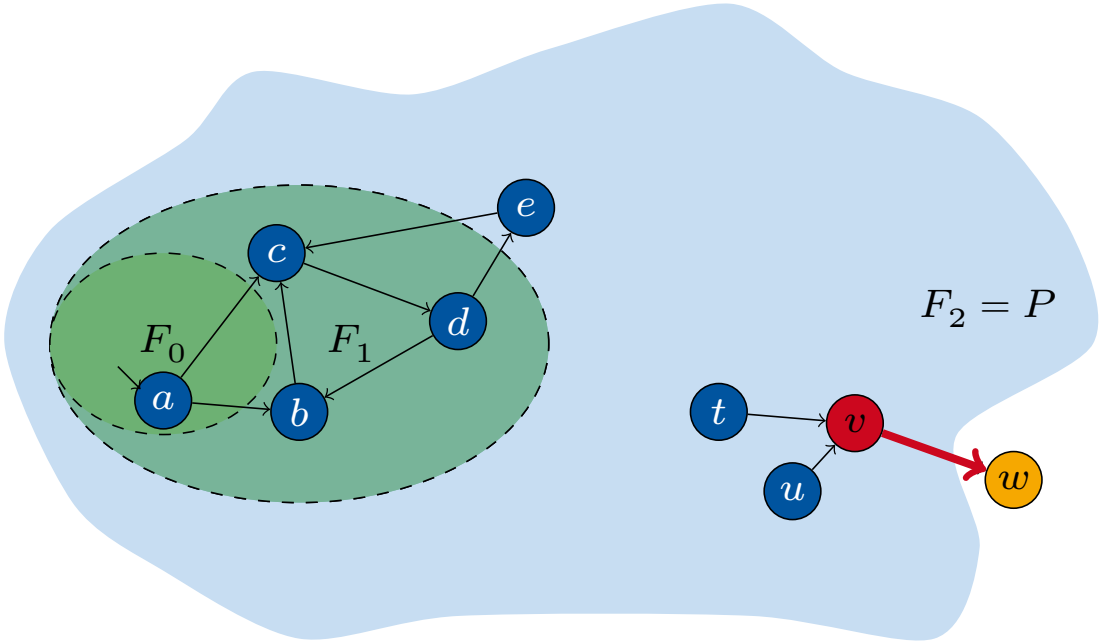

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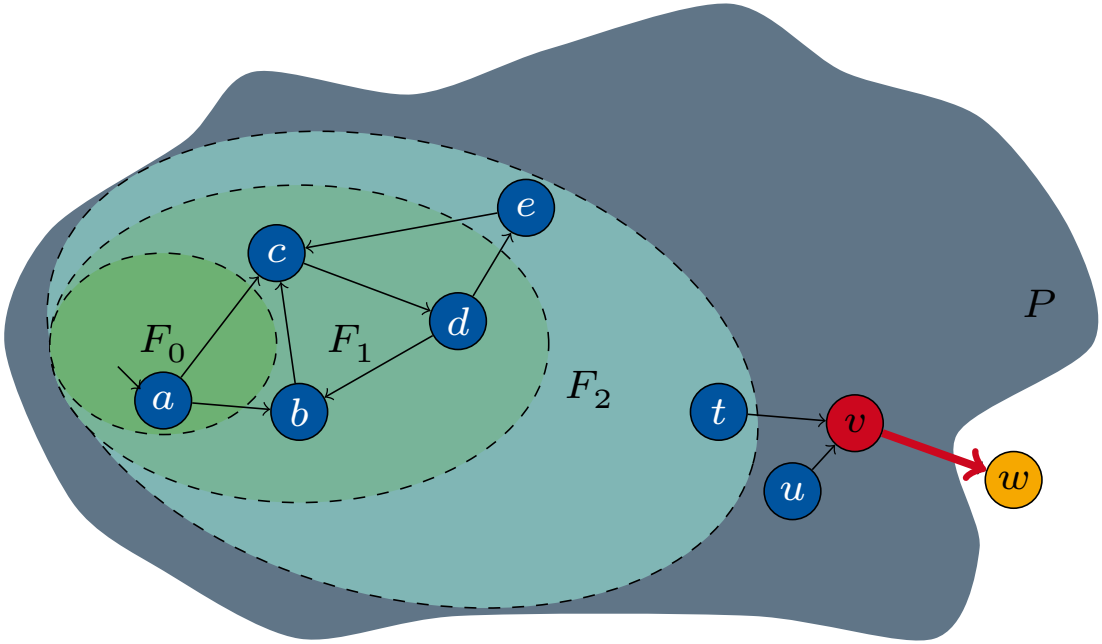

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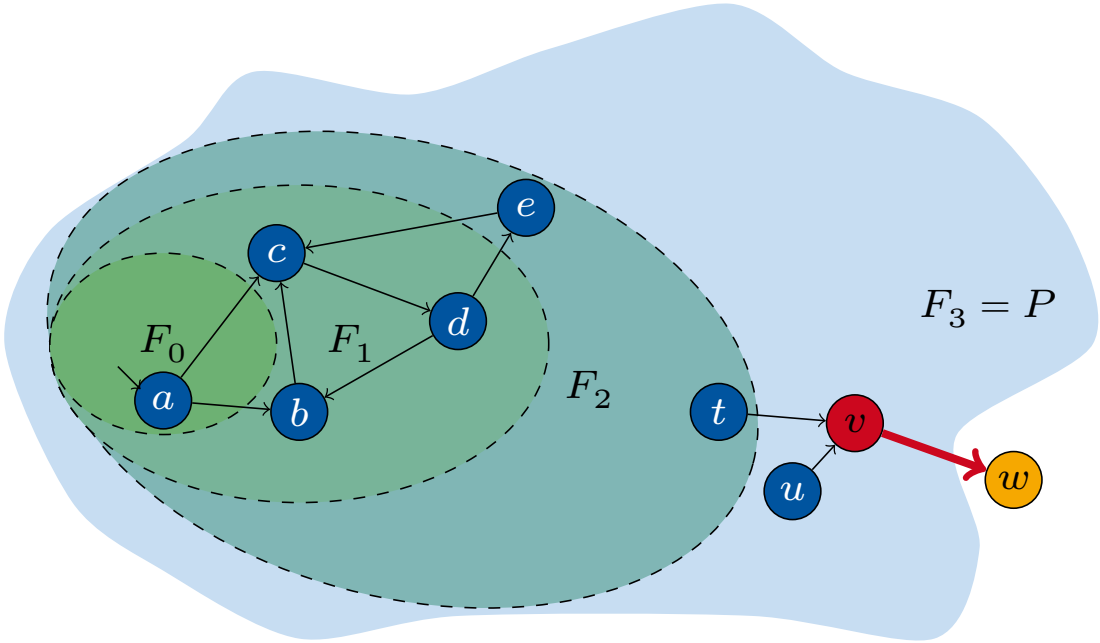

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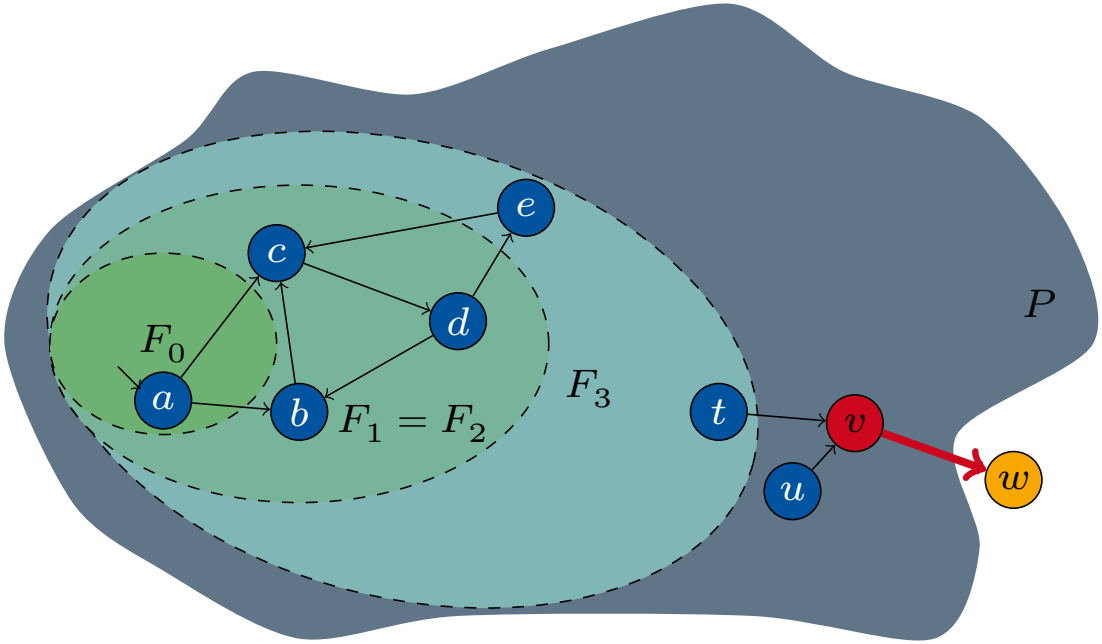

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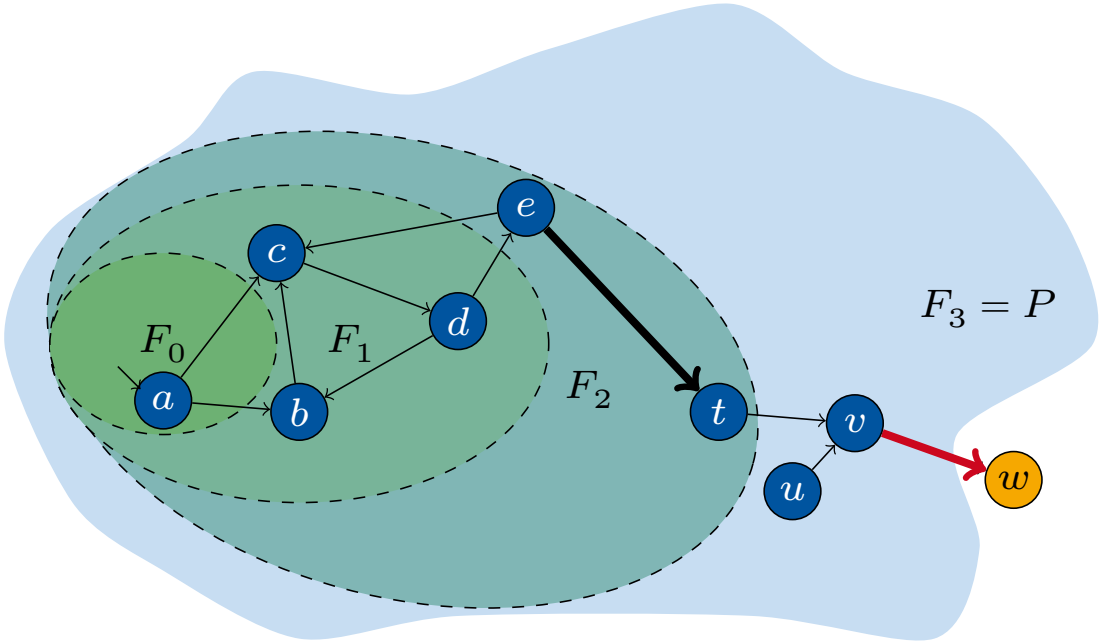

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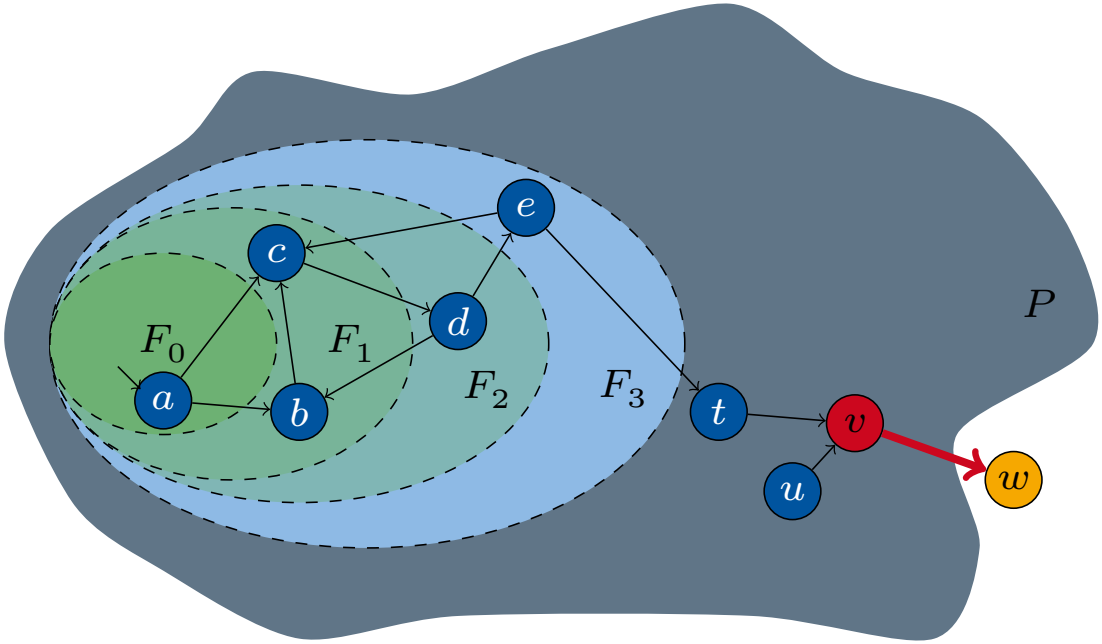

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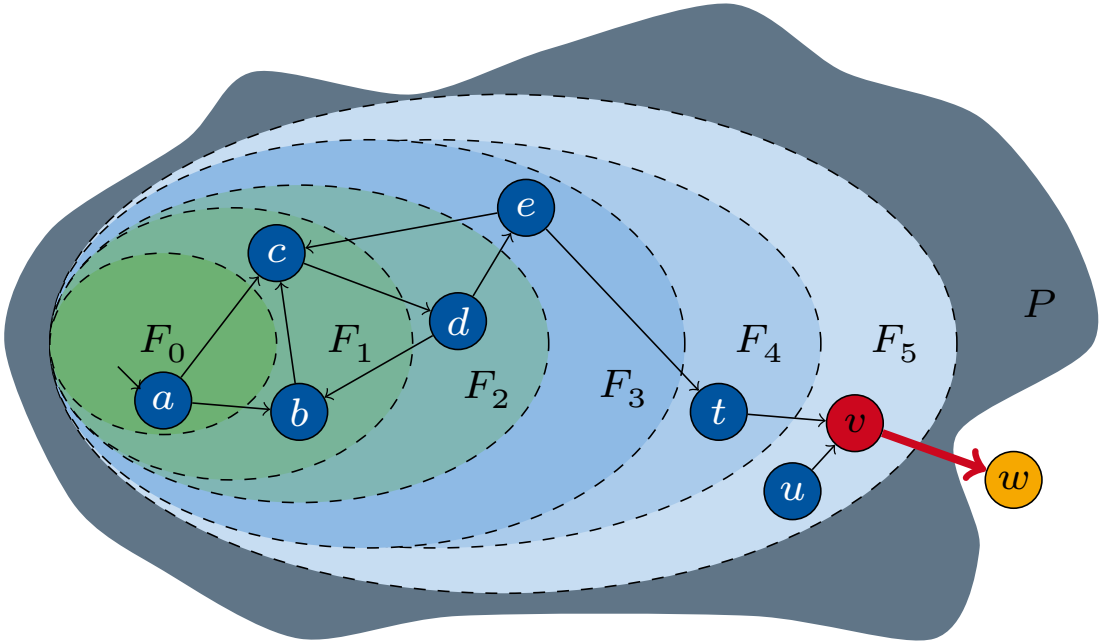

IC3 on Control Flow Automata

Motivation

Lifting to software model checking

- IC3 had a deep impact in hardware model checking
- Showed much better performance than known techniques
- Nowadays employed in most major hardware model checking tools

Challenges

- Domain in hardware model checking finite (bit-level)
- How to handle infinite state spaces?
- How to encode finite control flow?

IC3 on Control Flow Automata

Control Flow Automaton (CFA)

A CFA $\mathcal{A} = (L, G, l_0, l_E)$ consists of a set of *locations* $L = \{0, \dots, n\}$ and edges in $G \subseteq L \times QFFO \times L$ labeled with *quantifier-free* first-order formulas, an *initial location* l_0, and an *error location* l_E.

[1] CG12.

IC3 on Control Flow Automata

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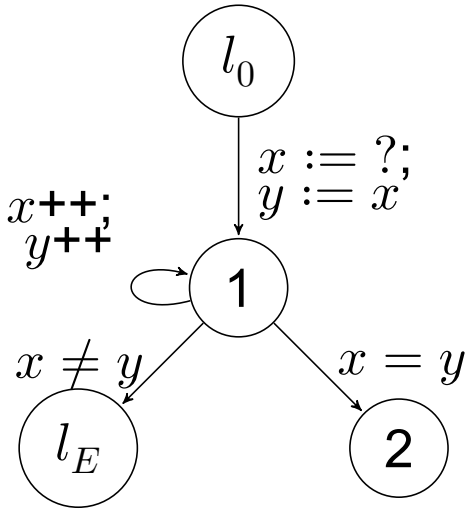
Idea

- Encoding of control flow using special pc variable *not efficient*[1]
- *Extraction* of control flow *advantageous*
- Instead of unrolling into ART [CG12] apply IC3 *directly on CFA*
- For *every location* in the CFA construct frames $F_0, \dots, F_k$
- Frames represent *overapproximations* of i-step reachability *in location*
- Explicit control flow locations allow to take *only single transitions* into account

[1] CG12.

IC3 on Control Flow Automata

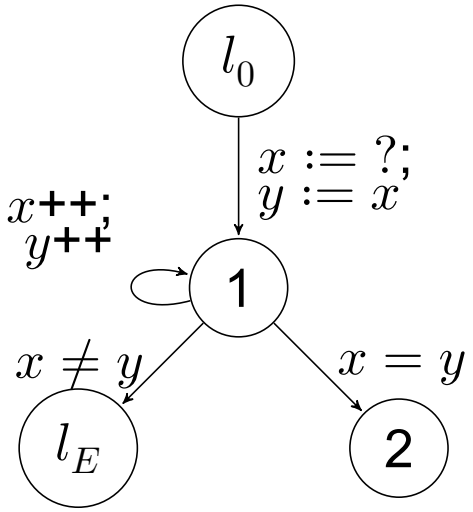
Example



Initial location: l_0
Error location: l_E
Terminating location: 2

IC3 on Control Flow Automata

Example



Frames $F_{(i,l)}$

i: \ l:	l_0	1
0	true	false
1	true	true

IC3 on Control Flow Automata

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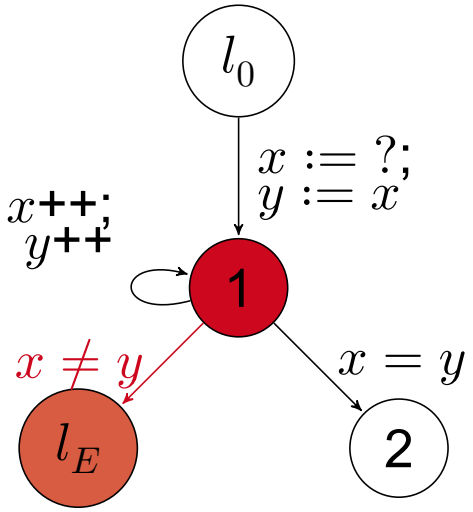

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CTI (1,$x \neq y$), level 1

IC3 on Control Flow Automata

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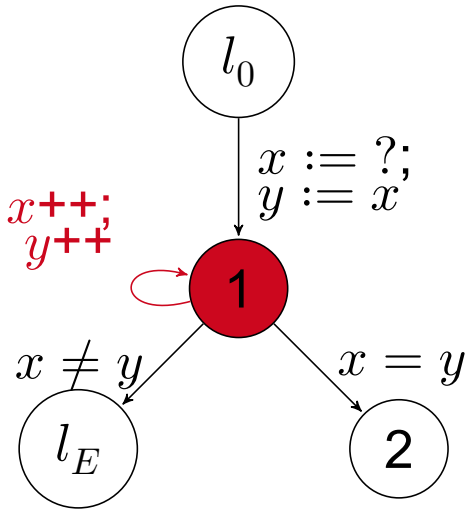

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CTI (1,$x \neq y$), level 1

$$SAT(F_{(0,1)} \wedge \neg(x \neq y) \wedge T_{1\to 1} \wedge x' \neq y')$$

IC3 on Control Flow Automata

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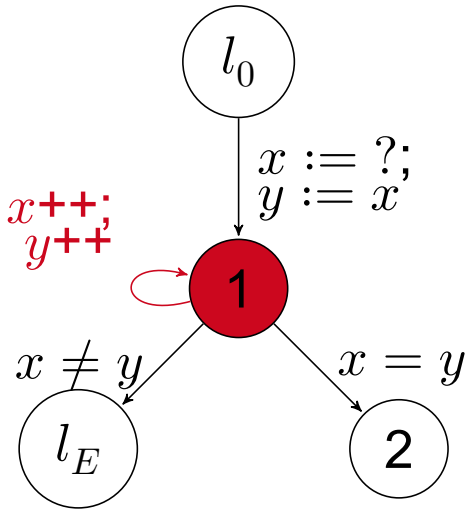

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IC3 on Control Flow Automata

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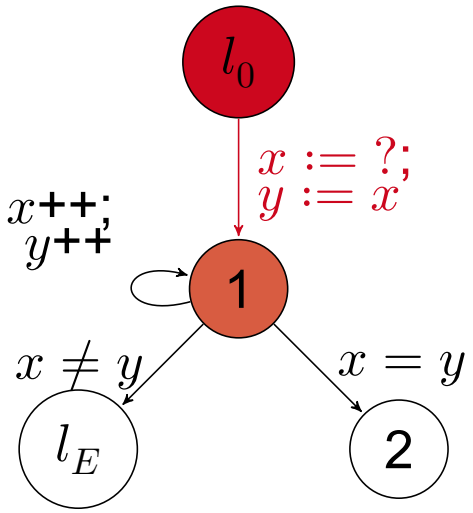

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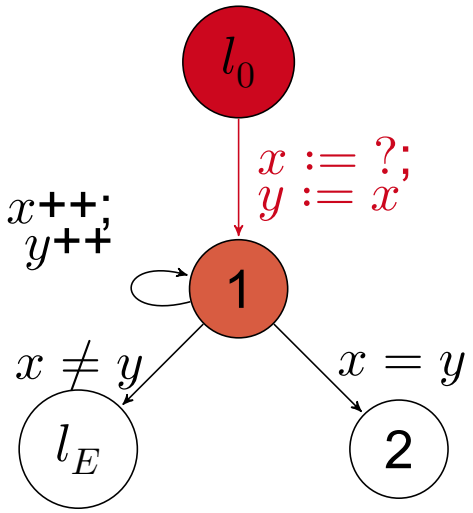

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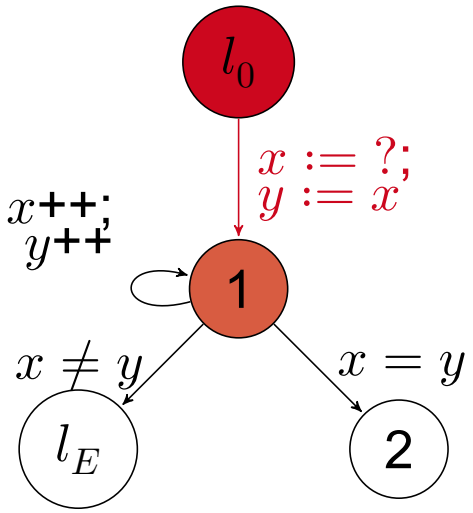

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0	true	false
1	true	$x = y$

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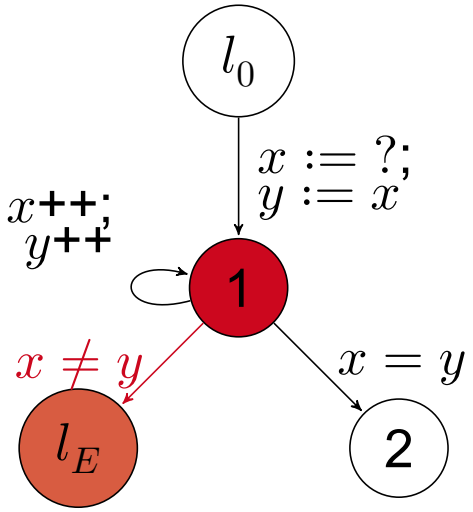

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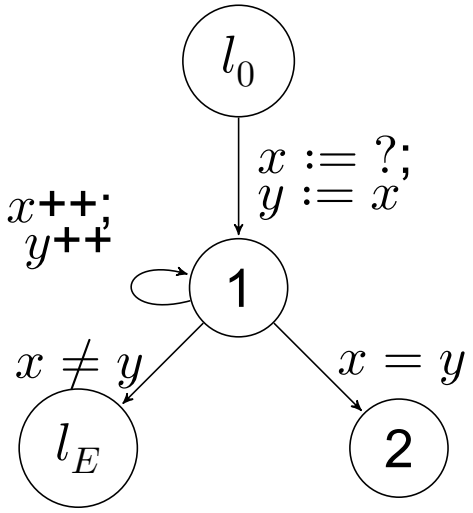

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References

- Aaron R. Bradley and Zohar Manna. “Checking Safety by Inductive Generalization of Counterexamples to Induction”. In: *FMCAD*. 2007, pp. 173–180.
- Aaron R. Bradley. “SAT-Based Model Checking without Unrolling”. In: *VMCAI*. 2011, pp. 70–87.
- Alessandro Cimatti and Alberto Griggio. “Software Model Checking via IC3”. In: *CAV*. 2012, pp. 277–293.
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