

9. Exercise sheet *Semantics and Verification of Software WS0809*

Due to Monday, 5 January 2009, *before* the exercise course begins.

Exercise 9.1:

(5 points)

Given the following program:

```
x := x + 1;
y := 1;
while x * y ≠ z do
  if x * y > z then
    x := x + 1;
    y := 1;
  else
    y := y + 1;
```

- Draw the control flow graph of this program;
- Perform an *available expression analysis* on this program.

Solution

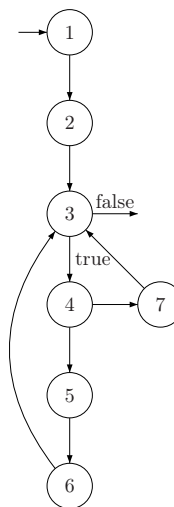
Computes the lowest prime factor x of z which is greater than the initial value of x . If there exists no such prime factor, x will be z and y will be 1.

Labeled:

```
[x := x + 1]1;
[y := 1]2;
while [x * y ≠ z]3 do
  if [x * y > z]4 then
    [x := x + 1]5;
    [y := 1]6;
  else
    [y := y + 1]7;
```

$AExp_c = \{x + 1, x * y, y + 1\}$

$l \in \mathbf{Lab}_c$	$kill_{AE}(B^l)$	$gen_{AE}(B^l)$
1	$\{x + 1, x * y\}$	\emptyset
2	$\{y + 1, x * y\}$	\emptyset
3	\emptyset	$\{x * y\}$
4	\emptyset	$\{x * y\}$
5	$\{x + 1, x * y\}$	\emptyset
6	$\{y + 1, x * y\}$	\emptyset
7	$\{y + 1, x * y\}$	\emptyset



$$\begin{aligned}
AE_1 &= \emptyset \\
AE_2 &= \varphi_1(AE_1) \\
&= \emptyset \\
AE_3 &= \varphi_2(AE_2) \cap \varphi_6(AE_6) \cap \varphi_7(AE_7) \\
&= \emptyset \\
AE_4 &= \varphi_3(AE_3) \\
&= \{x * y\} \\
AE_5 &= \varphi_4(AE_4) \\
&= \{x * y\} \\
AE_6 &= \varphi_5(AE_5) \\
&= \emptyset \\
AE_7 &= \varphi_4(AE_4) \\
&= \{x * y\}
\end{aligned}$$

□

Exercise 9.2:

(3 points)

A modification of the *available expressions analysis* detects when an expression is available in a *particular variable*: an expression a is available in x at label l if it has been evaluated and assigned to x on all paths leading to l and if the values of x and the variables in the expression have not changed since then. Develop the data flow equations for this analysis.

Solution

$AE'_l \subseteq \mathbf{AExp}_c \times (\mathbf{Var}_c \cup \{\cdot\})$ for each $l \in \mathbf{Lab}_c$

$$AE'_l = \begin{cases} \emptyset & \text{if } l = \text{init}(c) \\ \bigcap \{\varphi_{l'}(AE'_{l'}) \mid (l', l) \in \text{flow}(c)\} & \text{otherwise} \end{cases}$$

where $\varphi_{l'}(A) := (A \setminus \text{kill}_{AE'}(B^{l'})) \cup \text{gen}_{AE'}(B^{l'})$

$$\begin{aligned}
\text{kill}_{AE'}([\text{skip}]^l) &:= \emptyset \\
\text{kill}_{AE'}([x := a]^l) &:= \{(a', x') \in \mathbf{AExp}_c \times (\mathbf{Var}_c \cup \{\cdot\}) \mid x \in FV(a')\} \cup \{(a', x) \in \mathbf{AExp}_c \times (\mathbf{Var}_c \cup \{\cdot\})\} \\
\text{kill}_{AE'}([b]^l) &:= \emptyset \\
\text{gen}_{AE'}([\text{skip}]^l) &:= \emptyset \\
\text{gen}_{AE'}([x := a]^l) &:= \{(a, x) \in \mathbf{AExp}_c \times \mathbf{Var}_c \mid x \notin FV(a)\} \\
\text{gen}_{AE'}([b]^l) &:= \{(a, \cdot) \mid a \in \mathbf{AExp}_b\}
\end{aligned}$$

□

Exercise 9.3:

(4 points)

Give the following program:

```

y := 1;
while x > 0 do x := x - 1;
y := 2;

```

- Perform a *live variable analysis* on this program;
- Show that the solution of the equation system is not unique.

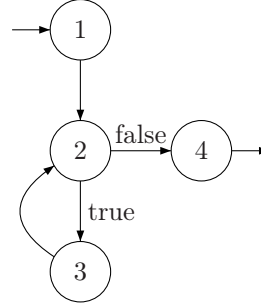
Solution

(a) Labeled:

```
[y := 1]1;
while [x > 0]2 do [x := x - 1]3;
[y := 2]4;
```

$\mathbf{Var}_c = \{x, y\}$

$l \in \mathbf{Lab}_c$	$kill_{LV}(B^l)$	$gen_{LV}(B^l)$
1	$\{y\}$	\emptyset
2	\emptyset	$\{x\}$
3	$\{x\}$	$\{x\}$
4	$\{y\}$	\emptyset



$$LV_4 = \{x, y\}$$

$$\begin{aligned} LV_3 &= \varphi_2(LV_2) \\ &= LV_2 \setminus \emptyset \cup \{x\} \\ &= LV_2 \cup \{x\} \end{aligned}$$

$$\begin{aligned} LV_2 &= \varphi_4(LV_4) \cup \varphi_3(LV_3) \\ &= (LV_4 \setminus \{y\}) \cup \emptyset \cup (LV_3 \setminus \{x\}) \cup \{x\} \\ &= LV_2 \cup \{x\} \end{aligned}$$

$$(lfp) = \{x\} \quad (\text{another solution would be } \{x, y\})$$

$$\begin{aligned} LV_1 &= \varphi_2(LV_2) \\ &= LV_2 \setminus \emptyset \cup \{x\} \\ &= LV_2 \cup \{x\} \end{aligned}$$

Solution:

$$LV_1 = LV_2 = LV_3 = \{x\}, \quad LV_4 = \{x, y\}$$

Dead code elimination yields:

```
while x > 0 do x := x - 1; y := 2
```

(b) Another solution is

$$LV_1 = LV_2 = LV_3 = LV_4 = \{x, y\}$$

□