Software Modeling and Verification
Lehrstuhl für Informatik 2
RWTH Aachen University

Prof. Dr. Ir. J.-P. Katoen
Priv.-Doz. Dr. T. Noll
T. Han, D. Klink

Modeling Concurrent and Probabilistic Systems

Summer Term 09

– Series 10 –

Hand in until July 23 before the exercise class.

Exercise 1 (3 points)

Show that:

$$\Big((P \oplus_{\frac{1}{2}} Q) \oplus_{\frac{1}{3}} (R \oplus_{\frac{3}{4}} P)\Big) \oplus_{\frac{2}{3}} Q \;\sim_p\; (Q \oplus_{\frac{2}{3}} P) \oplus_{\frac{2}{3}} R.$$

Solution The following equations hold:

$$\begin{aligned} P \oplus_p P &\sim_p P && (1)\\ P \oplus_p Q &\sim_p Q \oplus_{1-p} P && (2)\\ (P \oplus_p Q) \oplus_q R &\sim_p P \oplus_{p\cdot q} (Q \oplus_{\frac{q-p\cdot q}{1-p\cdot q}} R) && (3) \end{aligned}$$

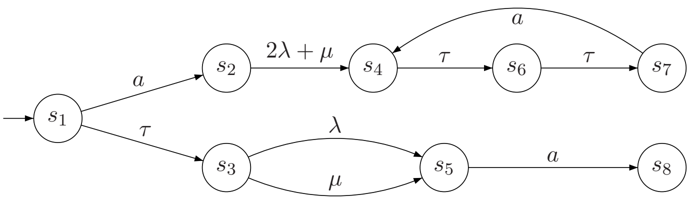
We can infer

$$\begin{aligned} & \Big((P \oplus_{\frac{1}{2}} Q) \oplus_{\frac{1}{3}} (R \oplus_{\frac{3}{4}} P)\Big) \oplus_{\frac{2}{3}} Q \\ \overset{(3)}{\sim}_p\ & (P \oplus_{\frac{1}{2}} Q) \oplus_{\frac{2}{9}} ((R \oplus_{\frac{3}{4}} P) \oplus_{\frac{4}{7}} Q) \\ \overset{(2)}{\sim}_p\ & (P \oplus_{\frac{1}{2}} Q) \oplus_{\frac{2}{9}} (Q \oplus_{\frac{3}{7}} (R \oplus_{\frac{3}{4}} P)) \\ \overset{(3)}{\sim}_p\ & ((P \oplus_{\frac{1}{2}} Q) \oplus_{\frac{2}{5}} Q) \oplus_{\frac{5}{9}} (R \oplus_{\frac{3}{4}} P) \\ \overset{(3)}{\sim}_p\ & (P \oplus_{\frac{1}{5}} (Q \oplus_{\frac{1}{4}} Q)) \oplus_{\frac{5}{9}} (R \oplus_{\frac{3}{4}} P) \\ \overset{(1)}{\sim}_p\ & (P \oplus_{\frac{1}{5}} Q) \oplus_{\frac{5}{9}} (R \oplus_{\frac{3}{4}} P) \\ \overset{(2)}{\sim}_p\ & (P \oplus_{\frac{1}{5}} Q) \oplus_{\frac{5}{9}} (P \oplus_{\frac{1}{4}} R) \\ \overset{(3)}{\sim}_p\ & ((P \oplus_{\frac{1}{5}} Q) \oplus_{\frac{5}{6}} P) \oplus_{\frac{2}{3}} R) \\ \overset{(2)}{\sim}_p\ & ((Q \oplus_{\frac{4}{5}} P) \oplus_{\frac{5}{6}} P) \oplus_{\frac{2}{3}} R) \\ \overset{(3)}{\sim}_p\ & (Q \oplus_{\frac{2}{3}} (P \oplus_{\frac{1}{2}} P)) \oplus_{\frac{2}{3}} R) \\ \overset{(1)}{\sim}_p\ & (Q \oplus_{\frac{2}{3}} P) \oplus_{\frac{2}{3}} R) \end{aligned}$$

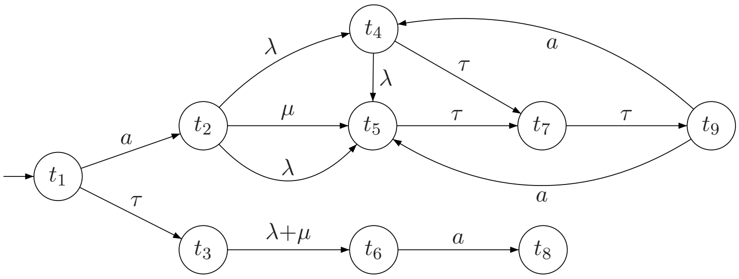
□

Exercise 2 (4 points)

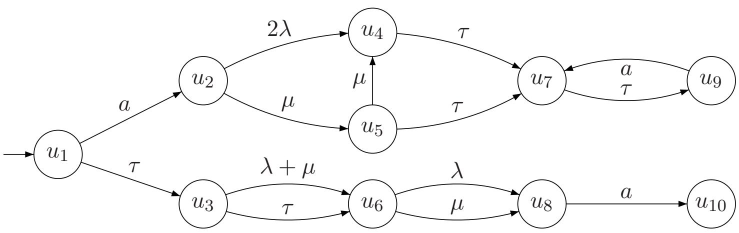
Given three IMCs E_1, E_2, E_3, where a, τ are actions, λ, μ are exponential rates.



IMC E_1



IMC E_2



IMC E_3

- Do we have $s_1 \sim_m t_1$?
- Do we have $t_1 \sim_m u_1$?
- Do we have $t_1 \approx_m u_1$?

Solution

- Yes. Due to the maximal progress assumption, the transition $t_4 \overset{\lambda}{\longmapsto} t_5$ will never happen. Thus, the equivalence induced by $R = \{(s_1, t_1),\ (s_2, t_2), (s_3, t_3),\ (s_4, t_4),\ (s_4, t_5), (s_5, t_6), (s_6, t_7), (s_7, t_9), (s_8, t_8)\}$ is the coarsest strong Markovian bisimulation of E_1 and E_2. $(s_1, t_1) \in R$, thus $s_1 \sim_m t_1$.

- No. Due to the maximal progress assumption, the transitions $u_3 \overset{\lambda+\mu}{\longmapsto} u_6$ and $u_5 \overset{\mu}{\longmapsto} u_4$ will never happen.

  $t_1 \overset{\tau}{\longrightarrow} t_3,\ t_3 \overset{\lambda+\mu}{\longmapsto} t_6;$

  $u_1 \overset{\tau}{\longrightarrow} u_3,\ u_3 \overset{\lambda+\mu}{\not\longmapsto}.$

  Thus, $t_1 \not\sim_m u_1$.

- Yes. The equivalence induced by $R' = \{(t_1, u_1),\ (t_2, u_2),\ (t_3, u_3),\ (t_3, u_6),\ (t_4, u_4),\ (t_5, u_5),\ (t_6, u_8),\ (t_7, u_7),\ (t_8, u_{10}),\ (t_9, u_9),\ (t_4, u_7),\ (u_4, t_7),\ (t_5, u_7),\ (u_5, t_7)\}$ is the coarsest weak Markovian bisimulation of E_2 and E_3. $(t_1, u_1) \in R'$, thus $t_1 \approx_m u_1$.

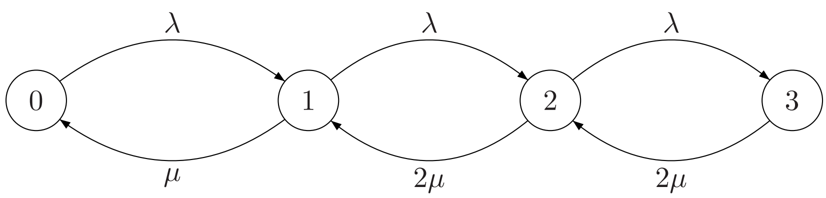
$\square$

Exercise 3 **(6 points)**

Given an M/M/2/1 queueing system as follows:

$$\begin{aligned} Arr &:= (\lambda).\alpha.Arr \\ Buff &:= \alpha.\delta.Buff \\ Proc &:= \delta.(\mu).Proc \\ Sys &:= \Big((Proc \parallel_{\varnothing} Proc) \parallel_{\{\delta\}} Buff\Big) \parallel_{\{\alpha\}} Arr \end{aligned}$$

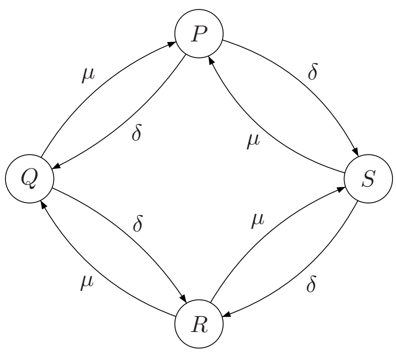
- Construct the IMC of Sys.
- Show that $Sys[f]$ with $f(\alpha) = \tau$ and $f(\delta) = \tau$ is weak Markovian bisimilar to:

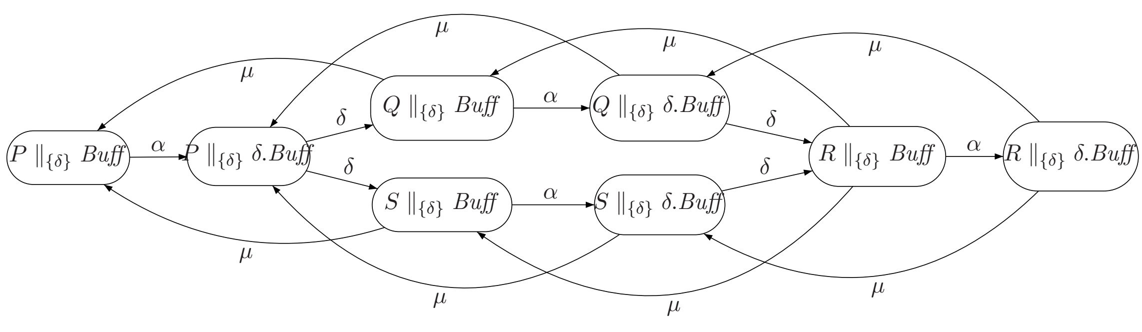


Solution

- Let $P = Proc \parallel_{\varnothing} Proc$, $Q = (\mu).Proc \parallel_{\varnothing} Proc$, $R = (\mu).Proc \parallel_{\varnothing} (\mu).Proc$, $S = Proc \parallel_{\varnothing} (\mu).Proc$.

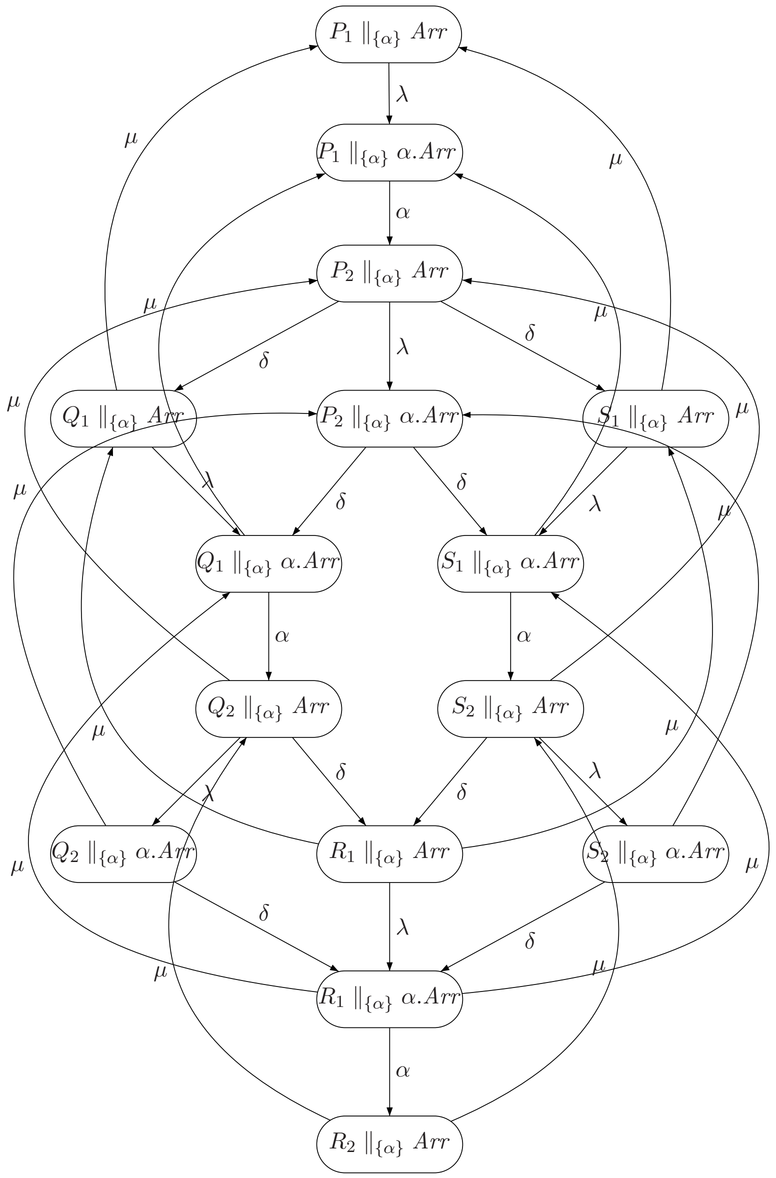
  The processes $Proc \parallel_{\varnothing} Proc$, $(Proc \parallel_{\varnothing} Proc) \parallel_{\{\delta\}} Buff$ and $\Big((Proc \parallel_{\varnothing} Proc) \parallel_{\{\delta\}} Buff\Big) \parallel_{\{\alpha\}} Arr$ are given as follows:





Let $P_1 = P \parallel_{\{\delta\}} Buff$, $P_2 = P \parallel_{\{\delta\}} \delta.Buff$, $Q_1 = Q \parallel_{\{\delta\}} Buff$, $Q_2 = Q \parallel_{\{\delta\}} \delta.Buff$,

$$R_1 = R \parallel_{\{\delta\}} Buff, \quad R_2 = R \parallel_{\{\delta\}} \delta.Buff, \quad S_1 = S \parallel_{\{\delta\}} Buff, \quad S_2 = S \parallel_{\{\delta\}} \delta.Buff.$$



- First replace all α- and δ-transitions with τ-transitions, which is the effect of the rename function $Sys[f]$.
- Then there are four equivalence classes under the weak bisimulation relation R:

$$C_0 = \{0,\ P_1 \parallel_{\{\alpha\}} Arr\},\ C_1 = \{1,\ P_1 \parallel_{\{\alpha\}} \alpha.Arr,\ P_2 \parallel_{\{\alpha\}} Arr,\ Q_1 \parallel_{\{\alpha\}} Arr,\ S_1 \parallel_{\{\alpha\}} Arr\},$$

$$C_2 = \{2,\ P_2 \parallel_{\{\alpha\}} Arr,\ Q_1 \parallel_{\{\alpha\}} \alpha.Arr,\ Q_2 \parallel_{\{\alpha\}} Arr,\ S_1 \parallel_{\{\alpha\}} \alpha.Arr, S_2 \parallel_{\{\alpha\}} Arr,\ R_1 \parallel_{\{\alpha\}} Arr\},$$

$$C_3 = \{3,\ R_1 \parallel_{\{\alpha\}} \alpha.Arr,\ Q_2 \parallel_{\{\alpha\}} \alpha.Arr,\ S_2 \parallel_{\{\alpha\}} \alpha.Arr,\ R_2 \parallel_{\{\alpha\}} Arr\}$$

- Check whether R is a weak bisimulation (Take C_2 for example):
  $(2, P_2 \parallel_{\{\alpha\}} \alpha.Arr) \in R$ because $2 \stackrel{\tau}{\not\longrightarrow}$ and $\mathbf{R}(2, C_3) = \delta$, $\mathbf{R}(2, C_1) = 2\mu$, then
  $P_2 \parallel_{\{\alpha\}} \alpha.Arr \Longrightarrow R_1 \parallel_{\{\alpha\}} \alpha.Arr$, and $\mathbf{R}(R_1 \parallel_{\{\alpha\}} \alpha.Arr, C_3) = \delta$; $\mathbf{R}(R_1 \parallel_{\{\alpha\}} \alpha.Arr, C_1) = 2\mu$.
  $(P_2 \parallel_{\{\alpha\}} \alpha.Arr, 2) \in R$ because $P_2 \parallel_{\{\alpha\}} \alpha.Arr \xrightarrow{\tau} Q_1 \parallel_{\{\alpha\}} \alpha.Arr$, and $(Q_1 \parallel_{\{\alpha\}} \alpha.Arr, 2) \in R$.

$\square$