Modeling Concurrent and Probabilistic Systems

Lecture 1: Introduction

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http://www-i2.informatik.rwth-aachen.de/i2/mcps07/

Winter Semester 2007/08
1 Preliminaries

2 Introduction

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(add “@cs.rwth-aachen.de” to e-mail addresses)
Target Audience

- Diploma programme (Informatik)
  - Theoretische Informatik
  - Vertiefungsfach Formale Methoden, Programmiersprachen und Softwarevalidierung

- Master programme (Software Systems Engineering)
  - Theoretical CS
  - Specialization Formal Methods, Programming Languages and Software Validation

In general:
- interest in formal models for software systems
- application of mathematical reasoning methods

Expected: basic knowledge in
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- mathematical logic
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- **Schedule:**
  - **Lecture** Tue 14:00–15:30 AH 2 (starting October 16)
  - **Lecture** Thu 13:30–15:00 AH 1 (starting November 8)
  - **Exercise class** Fri 10:00–11:30 AH 2 (starting October 26)

- see web page for single dates

- 1st assignment sheet: Fri Oct. 19 on web

- Work on assignments in groups of three

- **Examination** (8 ECTS credit points):
  - written or oral (depending on number of candidates);
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- Admission requires at least 50% of the points in the exercises

- Solutions to exercises and exam in English or German
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Motivation

Goal:

- describing and analyzing the behavior of concurrent and/or probabilistic systems

Motivation:

- supporting the design phase
  - “Programming Concurrent Systems”
    - synchronization, scheduling, fairness, absence of deadlocks, ...
- applying formal analysis methods
  - “Performance Modelling”
    - queue throughput, response time in real-time systems, ...
- verifying correctness properties
  - “Model Checking”
    - validation of mutual exclusion, fairness, no deadlocks, ...
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  - validation of mutual exclusion, fairness, no deadlocks, ...
Observation: concurrency introduces new phenomena

Example 1.1

\[
x := 0; \\
(x := x + 1 \parallel x := x + 2)
\]

value of \(x\): 3

- At first glance: \(x\) is assigned 3
- But: both parallel components could read \(x\) before it is written
- Thus: \(x\) is assigned 2,
- If exclusive access to shared memory and atomic execution of assignments guaranteed
  \[\implies\] only possible outcome: 3
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\[ x := 0; \]
\[ (x := x + 1 \parallel x := x + 2) \quad \text{value of } x: 23 \]
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\begin{array}{c}
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2
\end{array}
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- concurrency and
- interaction (here: via shared memory)

Conclusion

When modelling concurrent systems, the precise description of the mechanisms of both concurrency and interaction is crucially important.
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- **concurrency** and
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**Conclusion**

When modelling concurrent systems, the precise description of the mechanisms of both **concurrency** and **interaction** is crucially important.
Thus: “classical” model for sequential systems

$$\text{System : Input } \rightarrow \text{ Output}$$

(transformational systems) is not adequate

Missing: aspect of interaction

Rather: reactive systems which interact with environment and among themselves

Main interest: not terminating computations but infinite behavior (system maintains ongoing interaction with environment)

Examples:

- operating systems
- embedded systems controlling mechanical or electrical devices (planes, cars, home appliances, ...)
- power plants, production lines, ...
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- power plants, production lines, ...
Observation: reactive systems often safety critical
⇒ correct behavior has to be ensured

- Safety properties: “Nothing bad is going to happen.”
  E.g., “at most one process in the critical section”

- Liveness properties: “Eventually something good will happen.”
  E.g., “the server will finally answer”

- Fairness properties: “No component will starve to death.”
  E.g., “any process requiring entry to the critical section will eventually be admitted”
Our approach I

The formal verification of such properties requires a mathematical model of the underlying system. Here we use the following approach:

- **interaction** is interpreted by explicit, synchronous communication and
- **concurrency** is modelled by interleaving, i.e., the (communication) actions of concurrent processes are merged:

  \[(a; b) \parallel (x; y)\] corresponds to

  \[a \quad a \quad x\]
  \[b \quad x \quad a\]
  \[x \quad b \quad b\]
  \[y \quad y \quad y\]

  \[\Rightarrow\] reduction of concurrency to **nondeterminism**
  (cf. multitasking on sequential computers)

Possible alternatives:

- interaction via shared memory/asynchronous message passing/...
- concurrency via true parallelism (Petri Nets)
- later: probabilistic aspects [Katoen]
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\[(a; b) \parallel (x; y) \text{ corresponds to } a \quad a \quad x \quad b \quad or \quad x \quad b \quad or \quad x \quad a \quad or \quad ...
\]

\[\implies \text{reduction of concurrency to nondeterminism}
\]

(cf. multitasking on sequential computers)

Possible alternatives:

- interaction via shared memory/asynchronous message passing/...
- concurrency via true parallelism (Petri Nets)
- later: **probabilistic** aspects [Katoen]
“Primary meaning” of a system: potential of communication
i.e., the set of possible communication sequences

In particular:

- I/O modelled as communication with environment
- storage access modelled as communication with a “storage process”
Overview of the Course

1st part of course (CCS):

2. Calculus of Communicating Systems (CCS)
   (syntax, labeled transition systems, transition rules)
3. Equivalence of CCS Processes
   (trace equivalence, strong/weak bisimulation, observation congruence, axiomatizability of equivalences)
4. Case Study: Alternating-Bit Protocol
   (modeling channels/sender/receiver, correctness, extensions)

2nd part of course (Probabilistic Models):

5. Stochastic processes
   (Markov chains and decision processes)
6. Probabilistic (bi)simulation
   (strong bisimulation/simulation, simulation equivalence)
7. Probabilistic process algebra
   (probabilistic transition systems, operators, axiomatizability of probabilistic bisimulation)
8. Further Issues
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Literature

(also see the collection [“Handapparat Probabilistic Models for Concurrency / PMC”] at the CS Library)

- 1st part of course (CCS):
  - R. Milner: *Communication and Concurrency*
    Prentice-Hall, 1989
  - R. Milner: *Communicating and Mobile Systems: the π-calculus*
    Cambridge University Press, 1999
  - J.A. Bergstra, A. Ponse, S.A. Smolka: *Handbook of Process Algebra*
    Elsevier, 2001

- 2nd part of course (Probabilistic Models):
  - H.C. Tijms: *A first course in stochastic models*
    Wiley, 2003
  - J. Hillston: *A Compositional Approach to Performance Modelling*
    Cambridge University Press, 1996
  - H. Hermanns: *Interactive Markov Chains: The Quest for Quantified Quality*
    LNCS 2428, Springer, 2002
Outline

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History of CCS

- Robin Milner: *A Calculus of Communicating Systems*
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- Robin Milner: *Communication and Concurrency*
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**Approach:** describing concurrency on a simple and abstract level, using only a few basic primitives
- no explicit storage (variables)
- no explicit representation of values (numbers, Booleans, ...)
  
  → abstraction of communication potential of a concurrent system
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**Approach:** describing concurrency on a simple and abstract level, using only a few basic primitives
  - no explicit storage (variables)
  - no explicit representation of values (numbers, Booleans, ...)

$\Rightarrow$ abstraction of *communication potential* of a concurrent system
Definition 1.2 (Syntax of CCS)

Let $N$ be a set of (action) names.
- $\overline{N} := \{\overline{a} \mid a \in N\}$ denotes the set of co-names.
- $Act := N \cup \overline{N} \cup \{\tau\}$ is the set of actions where $\tau$ denotes the silent (or: unobservable) action.

Let $Pid$ be a set of process identifiers.

The set $Prc$ of process expressions is defined by the following syntax:

$$P ::= \text{nil} \quad \text{(inaction)}$$

$$| \alpha.P \quad \text{(prefixing)}$$

$$| P_1 + P_2 \quad \text{(choice)}$$

$$| P_1 \parallel P_2 \quad \text{(parallel composition)}$$

$$| \text{new } a \ P \quad \text{(restriction)}$$

$$| A(a_1, \ldots, a_n) \quad \text{(process call)}$$

where $\alpha \in Act$, $a, a_i \in N$, and $A \in Pid$. 
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where $\alpha \in Act$, $a, a_i \in N$, and $A \in Pid$. 
Definition 1.2 (continued)

A (recursive) process definition is an equation system of the form

\[(A_i(a_{i1}, \ldots, a_{in_i}) = P_i \mid 1 \leq i \leq k)\]

where \(k \geq 1\), \(A_i \in \text{Pid}\) (pairwise different), \(a_{ij} \in N\), and \(P_i \in \text{Prc}\) (with process identifiers from \(\{A_1, \ldots, A_k\}\)).
Meaning of CCS Constructs

- nil is an inactive process that can do nothing.
- $\alpha.P$ can execute $\alpha$ and then behaves as $P$.
- An action $a \in N$ ($\overline{a} \in \overline{N}$) is interpreted as an input (output, resp.) operation. Both are complementary: if executed in parallel (i.e., in $P_1 \parallel P_2$), they are merged into a $\tau$-action.
- $P_1 + P_2$ represents the non-deterministic choice between $P_1$ and $P_2$.
- $P_1 \parallel P_2$ denotes the concurrent execution of $P_1$ and $P_2$, involving interleaving or communication.
- The restriction $\text{new } a P$ declares $a$ as a local name which is only known in $P$.
- The behavior of a process call $A(a_1, \ldots, a_n)$ is defined by the right-hand side of the corresponding equation where $a_1, \ldots, a_n$ replace the formal name parameters.
Meaning of CCS Constructs

- nil is an **inactive process** that can do nothing.
- \( \alpha.P \) can execute \( \alpha \) and then behaves as \( P \).
- An action \( a \in N \) (\( \overline{a} \in \overline{N} \)) is interpreted as an **input** (output, resp.) operation. Both are complementary: if executed in parallel (i.e., in \( P_1 \parallel P_2 \)), they are merged into a \( \tau \)-action.
- \( P_1 + P_2 \) represents the **non-deterministic choice** between \( P_1 \) and \( P_2 \).
- \( P_1 \parallel P_2 \) denotes the **concurrent execution** of \( P_1 \) and \( P_2 \), involving interleaving or communication.
- The **restriction** \( \text{new} \ a \ P \) declares \( a \) as a local name which is only known in \( P \).
- The behavior of a **process call** \( A(a_1, \ldots, a_n) \) is defined by the right-hand side of the corresponding equation where \( a_1, \ldots, a_n \) replace the formal name parameters.
Meaning of CCS Constructs

- **nil** is an **inactive process** that can do nothing.
- **α.P** can execute **α** and then behaves as **P**.
- An action **a ∈ N** (**ā ∈ N̅**) is interpreted as an **input** (**output**, resp.) operation. Both are complementary: if executed in parallel (i.e., in **P₁ || P₂**), they are merged into a **τ**-action.
- **P₁ + P₂** represents the **non-deterministic choice** between **P₁** and **P₂**.
- **P₁ || P₂** denotes the **concurrent execution** of **P₁** and **P₂**, involving **interleaving** or **communication**.
- The **restriction** **new a P** declares **a** as a local name which is only known in **P**.
- The behavior of a **process call** **A(a₁, . . . , aₙ)** is defined by the right-hand side of the corresponding equation where **a₁, . . . , aₙ** replace the formal name parameters.
nil is an inactive process that can do nothing.

\( \alpha.P \) can execute \( \alpha \) and then behaves as \( P \).

An action \( a \in N \) (\( \overline{a} \in \overline{N} \)) is interpreted as an input (output, resp.) operation. Both are complementary: if executed in parallel (i.e., in \( P_1 \parallel P_2 \)), they are merged into a \( \tau \)-action.

\( P_1 + P_2 \) represents the non-deterministic choice between \( P_1 \) and \( P_2 \).

\( P_1 \parallel P_2 \) denotes the concurrent execution of \( P_1 \) and \( P_2 \), involving interleaving or communication.

The restriction new \( a \) \( P \) declares \( a \) as a local name which is only known in \( P \).

The behavior of a process call \( A(a_1, \ldots, a_n) \) is defined by the right-hand side of the corresponding equation where \( a_1, \ldots, a_n \) replace the formal name parameters.
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An action \( a \in N (\overline{a} \in \overline{N}) \) is interpreted as an input (output, resp.) operation. Both are complementary: if executed in parallel (i.e., in \( P_1 \parallel P_2 \)), they are merged into a \( \tau \)-action.

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### Example 1.3

1. One-place buffer
2. Two-place buffer
3. Parallel specification of two-place buffer

(on the board)
Notational Conventions

- \( \bar{a} \) means \( a \)

- \( P_1 + \ldots + P_n \) (\( n \in \mathbb{N} \)) sometimes written as \( \sum_{i=1}^{n} P_i \) where \( \sum_{i=1}^{0} P_i := \text{nil} \)

- “.nil” can be omitted: \( a.b \) means \( a.b.\text{nil} \)

- new \( a, b \ P \) means new \( a \) new \( b \ P \)

- \( A(a_1, \ldots, a_n) \) sometimes written as \( A(\bar{a}) \), \( A() \) as \( A \)

- prefixing and restriction binds stronger than composition, composition binds stronger than choice:

\[
\text{new } a \ P + b.Q \parallel R \quad \text{means} \quad (\text{new } a \ P) + ((b.Q) \parallel R)
\]
Notational Conventions

- $\overparen{a}$ means $a$
- $P_1 + \ldots + P_n$ ($n \in \mathbb{N}$) sometimes written as $\sum_{i=1}^{n} P_i$ where $\sum_{i=1}^{0} P_i \equiv \text{nil}$
- "nil" can be omitted: $a.b$ means $a.b.nil$
- new $a$, $b$ $P$ means new $a$ new $b$ $P$
- $A(a_1, \ldots, a_n)$ sometimes written as $A(\overparen{a})$, $A()$ as $A$
- prefixing and restriction binds stronger than composition, composition binds stronger than choice:

\[
\text{new } a \ P + b.\ Q \parallel R \quad \text{means} \quad (\text{new } a \ P) + ((b.\ Q) \parallel R)
\]
Notational Conventions

- $\overline{a}$ means $a$

- $P_1 + \ldots + P_n$ ($n \in \mathbb{N}$) sometimes written as $\sum_{i=1}^{n} P_i$ where $\sum_{i=1}^{0} P_i := \text{nil}$

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\text{new } a \ P + b.Q \ || \ R \quad \text{means} \quad (\text{new } a \ P) + ((b.Q) \ || \ R)
\]
Notational Conventions

- $\overline{a}$ means $a$

- $P_1 + \ldots + P_n \ (n \in \mathbb{N})$ sometimes written as $\sum_{i=1}^{n} P_i$ where $\sum_{i=1}^{0} P_i := \text{nil}$

- “.nil” can be omitted: $a.b$ means $a.b.\text{nil}$

- new $a, b \ P$ means new $a$ new $b \ P$

- $A(a_1, \ldots, a_n)$ sometimes written as $A(\overline{a})$, $A()$ as $A$

- prefixing and restriction binds stronger than composition, composition binds stronger than choice:

  new $a \ P + b.Q \parallel R$ means $(\text{new } a \ P) + ((b.Q) \parallel R)$
Notational Conventions

- $\overline{a}$ means $a$

- $P_1 + \ldots + P_n$ ($n \in \mathbb{N}$) sometimes written as $\sum_{i=1}^{n} P_i$ where $\sum_{i=1}^{0} P_i := \text{nil}$

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- $A(a_1, \ldots, a_n)$ sometimes written as $A(\overline{a})$, $A()$ as $A$

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