Lazy abstractions for timed automata

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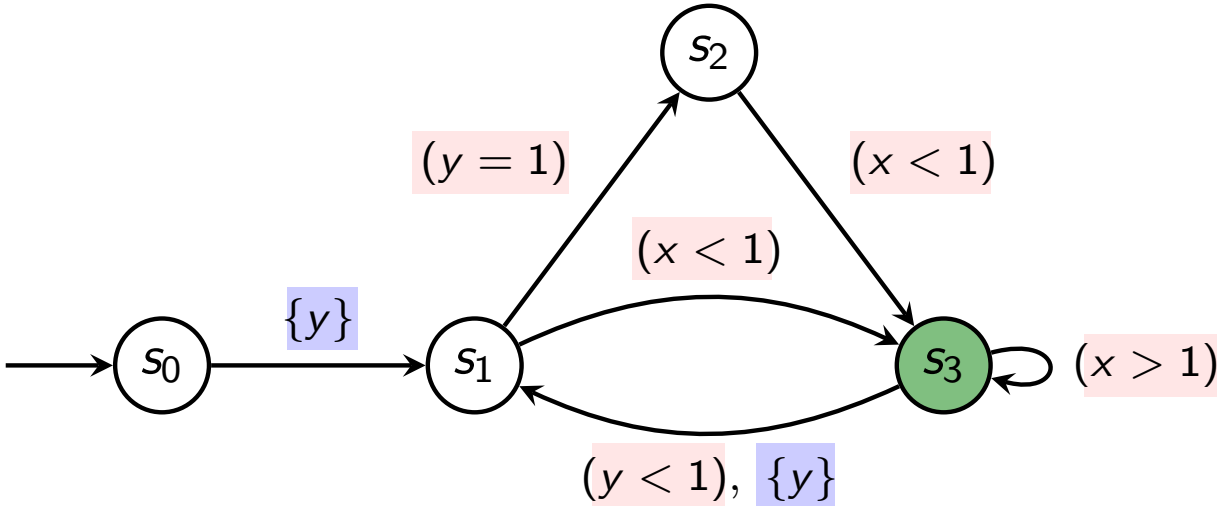
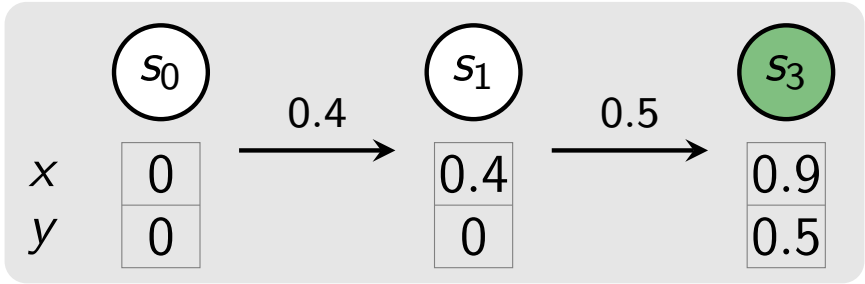
MOVES Seminar, March 2013

Timed Automata [AD94]



Run: finite sequence of transitions



- **accepting** if ends in green state

The problem we are interested in ...

Given a TA, does there **exist** an **accepting run**?

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Theorem [AD94]

This problem is **PSPACE-complete**

first solution based on Regions

Regions

Maximal bounds: $M : X \mapsto \mathbb{N} \cup \{-\infty\}$

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$$M(x) = 3,\ M(y) = 2$$

$$x \lessdot 0,\ x \gtrdot 0,\ x \lessdot 1,\ x \gtrdot 1,\ \ldots,\ x \lessdot 3,\ x \gtrdot 3$$
$$y \lessdot 0,\ y \gtrdot 0,\ \ldots,\ y \lessdot 2,\ y \gtrdot 2$$

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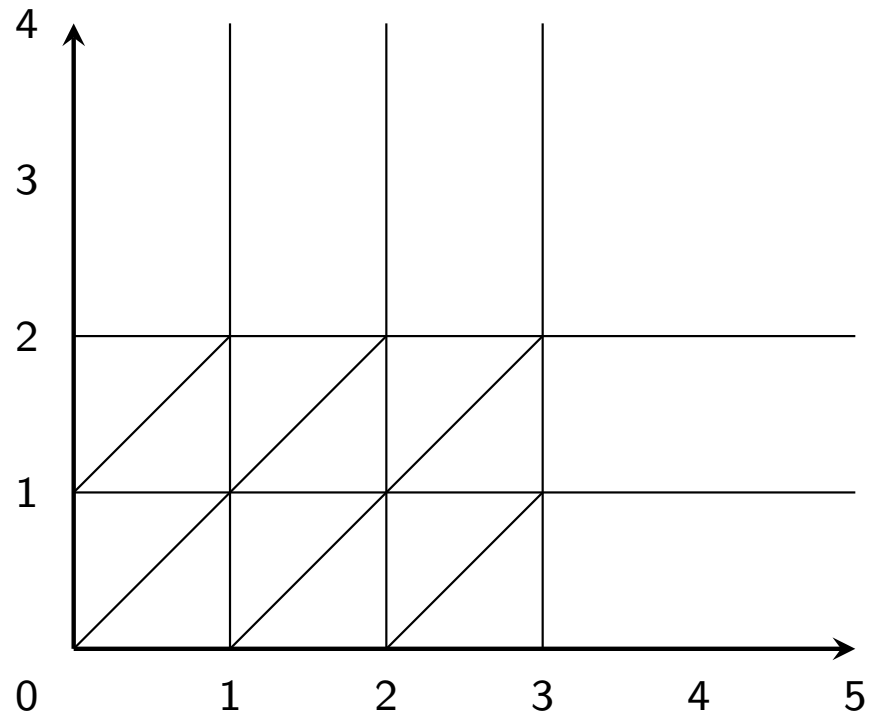
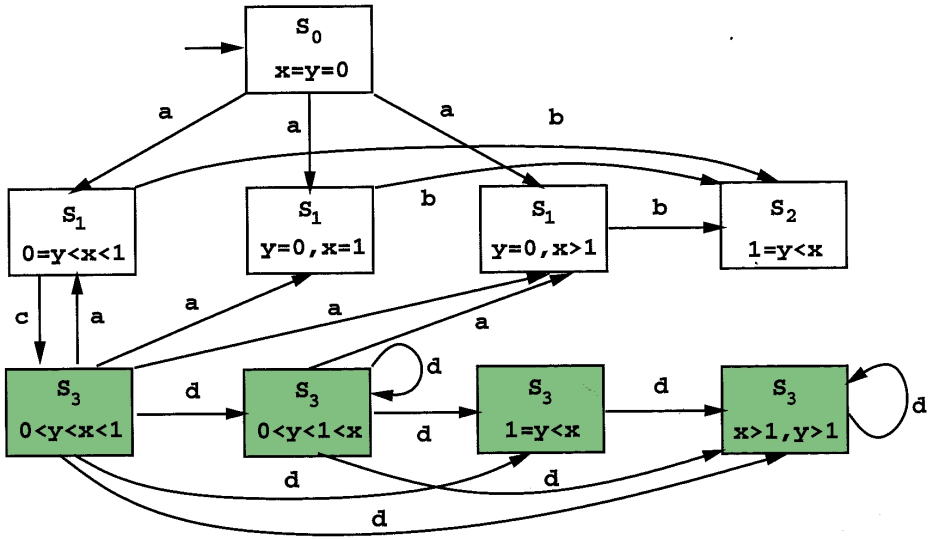
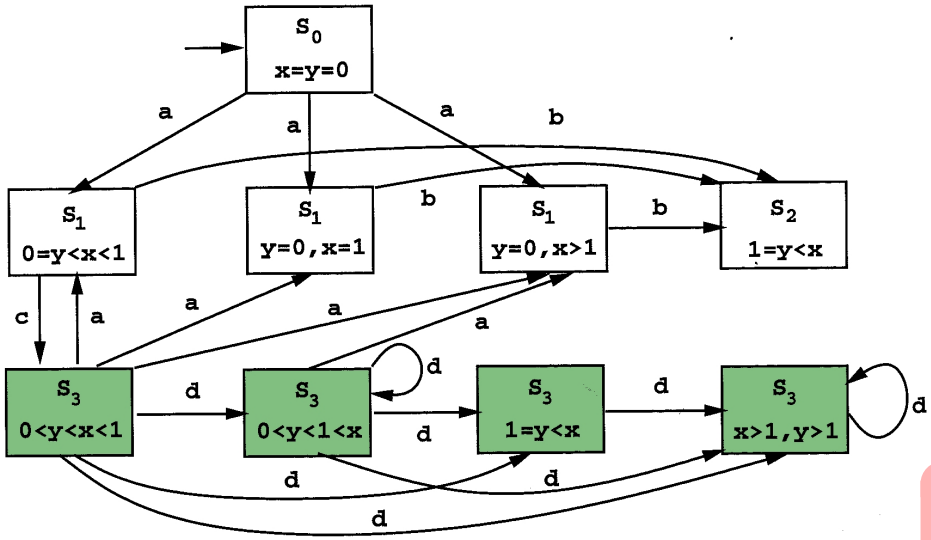




- Region: set of valuations satisfying the **same guards** w.r.t. time
- Finiteness: Parametrized by **maximal constant**

Sound and complete [AD94]

Region graph preserves state **reachability**



- Region: set of valuations satisfying the **same guards** w.r.t. time
- Finiteness: Parametrized by **maximal constant**

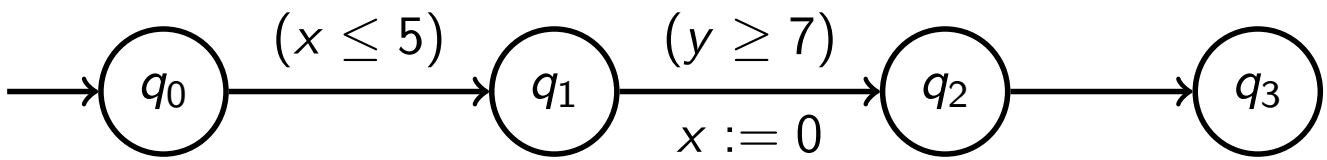
$\mathcal{O}(|X|!.M^{|X|})$ many regions!

Sound and complete [AD94]

Region graph preserves state **reachability**

A more efficient solution...

Key idea: Maintain **all valuations** reachable along a path



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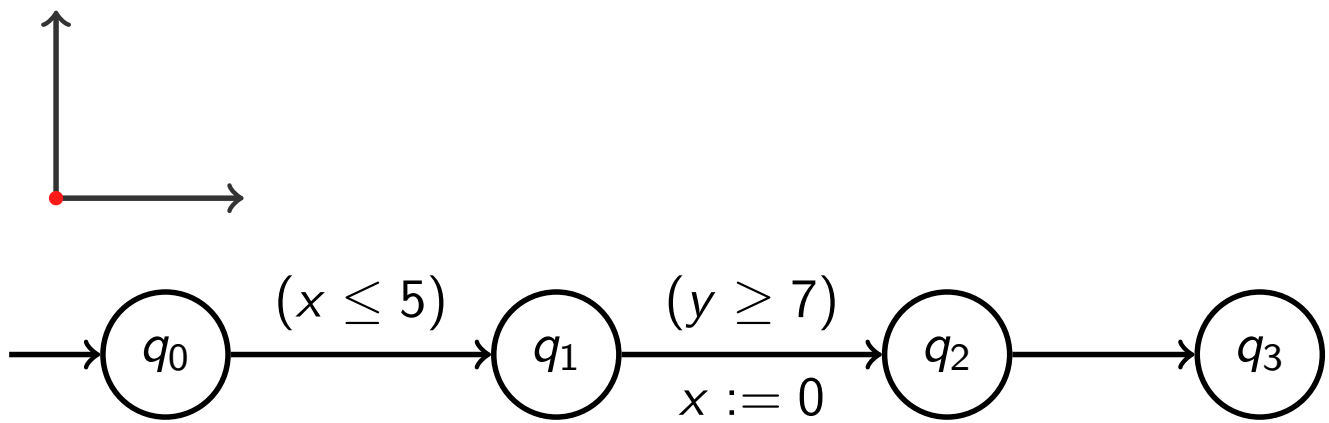

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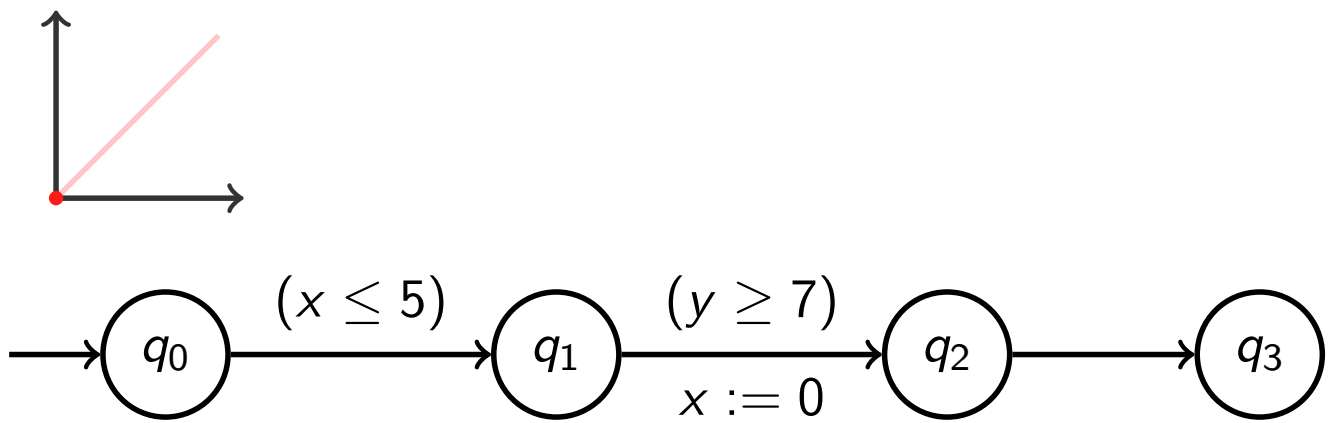

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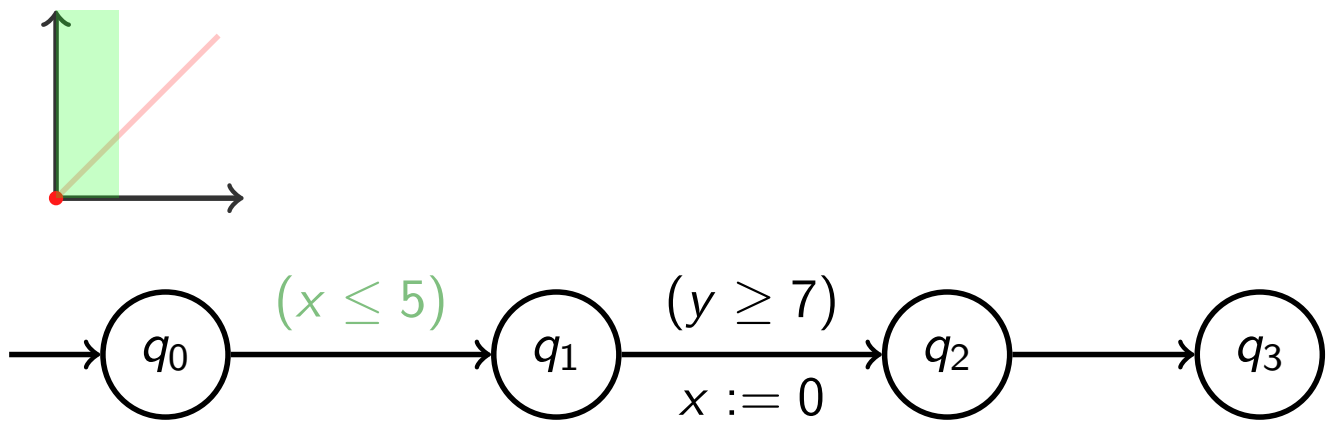

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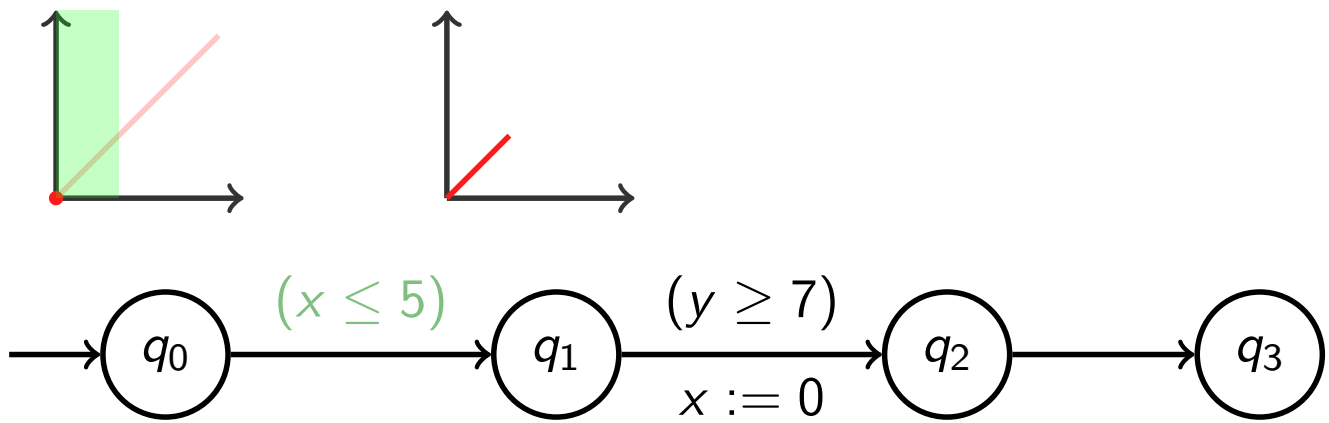

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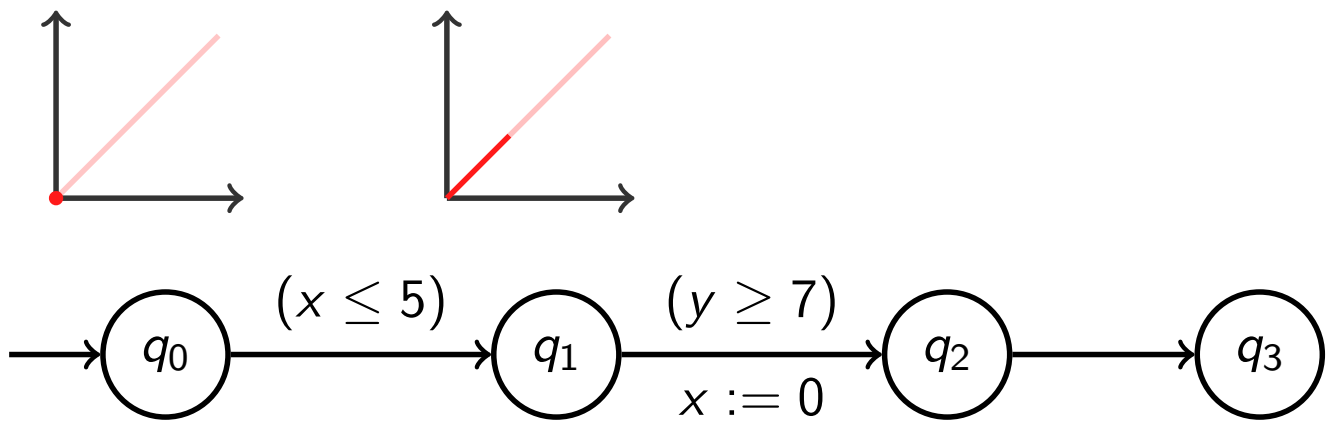

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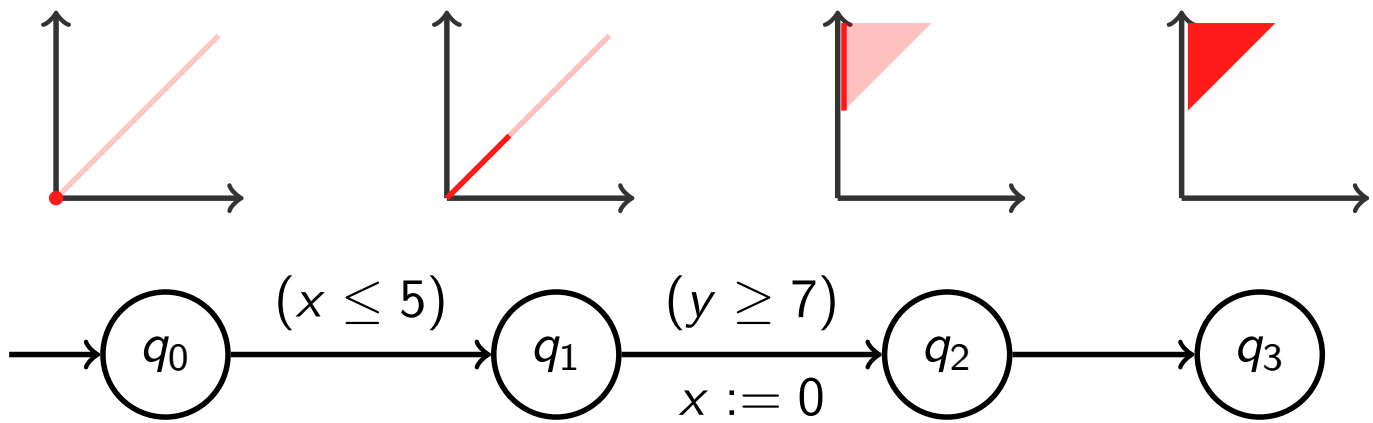

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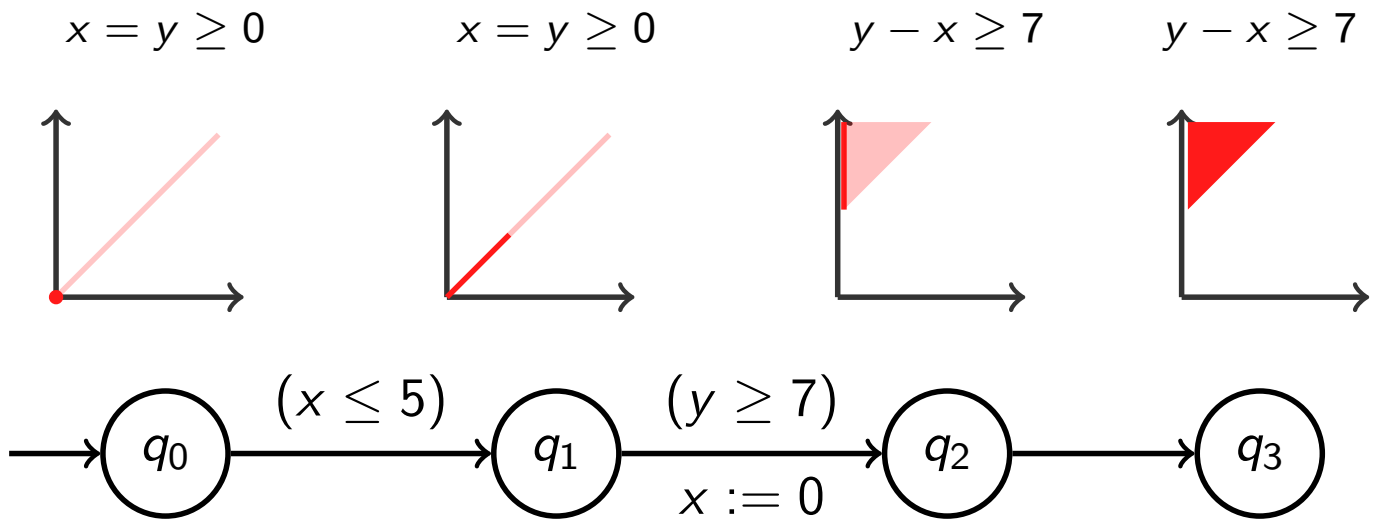
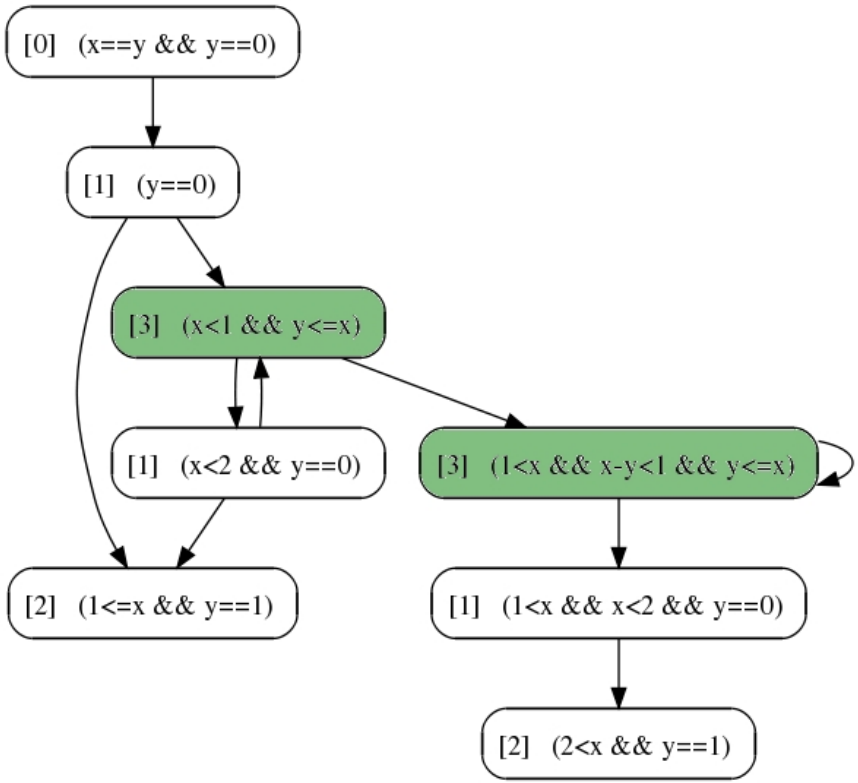


Zones and zone graph

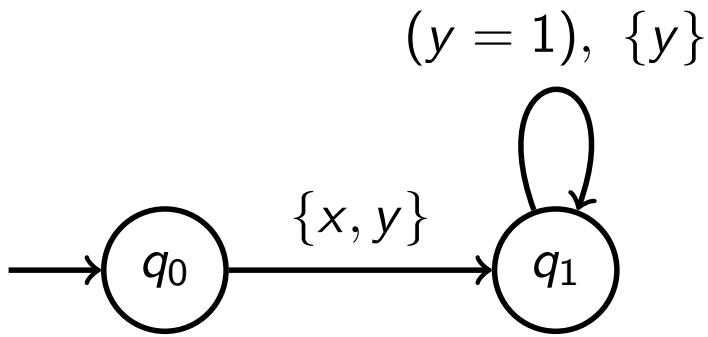


- Zone: set of valuations defined by conjunctions of constraints:
  - $x \sim c$
  - $x - y \sim c$
  - e.g. $(x - y \geq 1) \wedge y < 2$
- Representation: by DBM

Sound and complete [DT98]

Zone graph preserves state **reachability**

But the zone graph could be infinite ...



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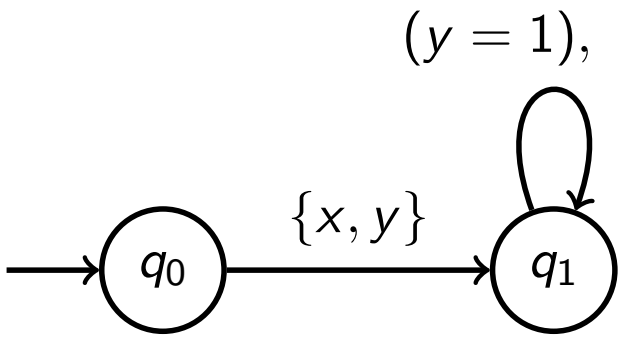

$(q_0, x - y = 0)$

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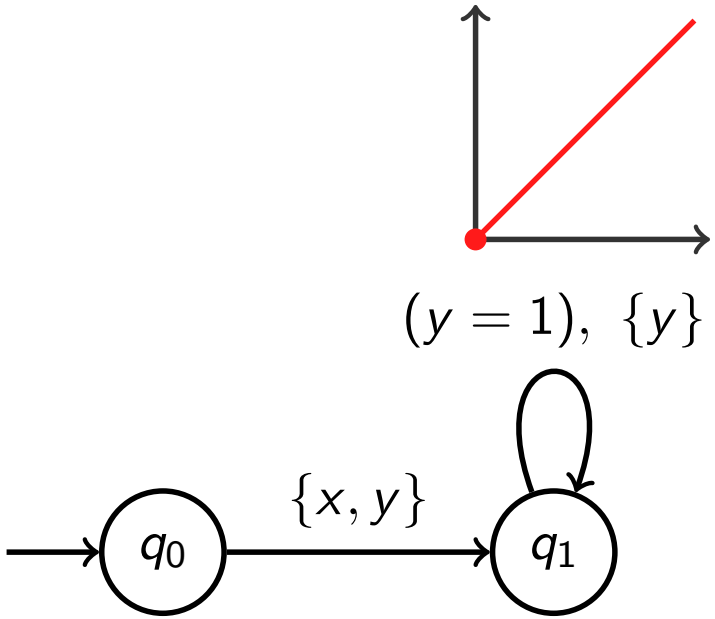

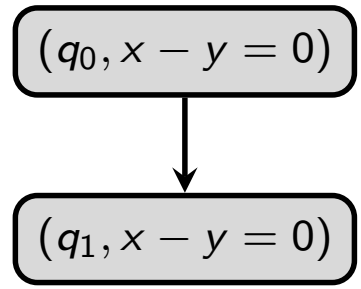

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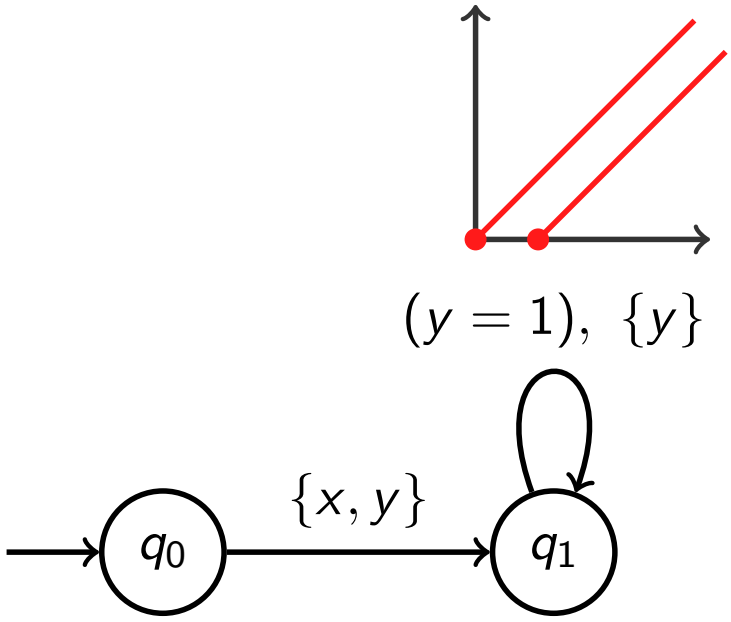

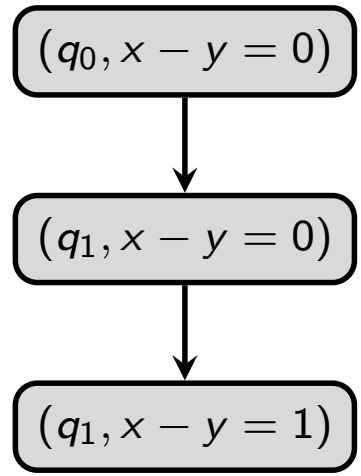

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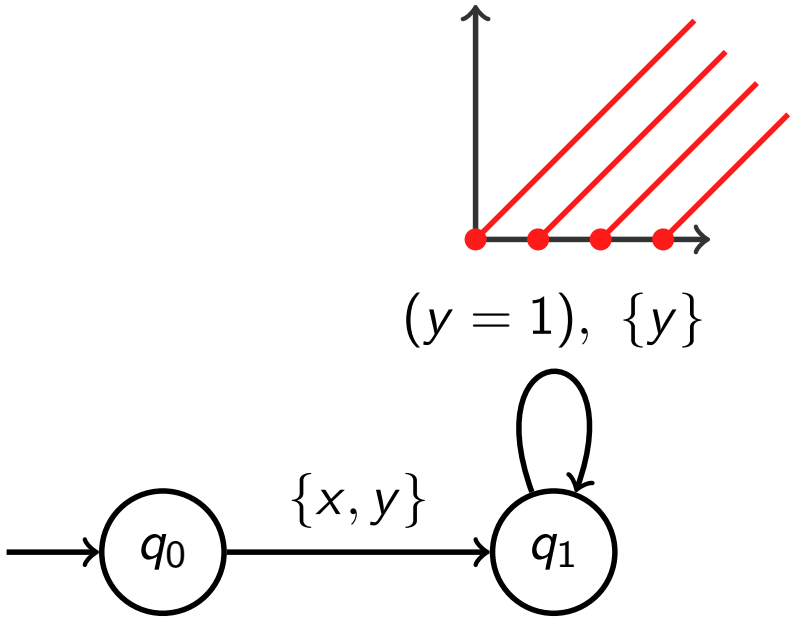

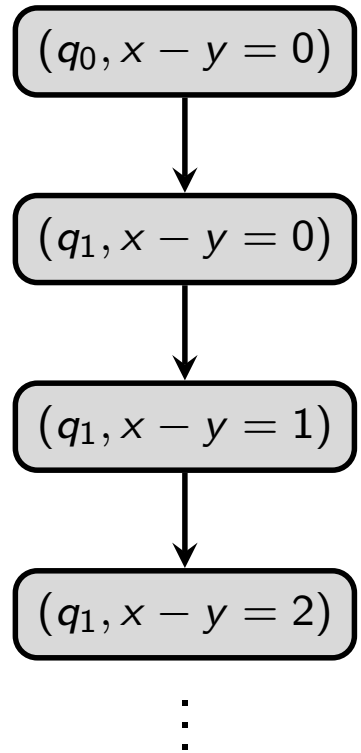

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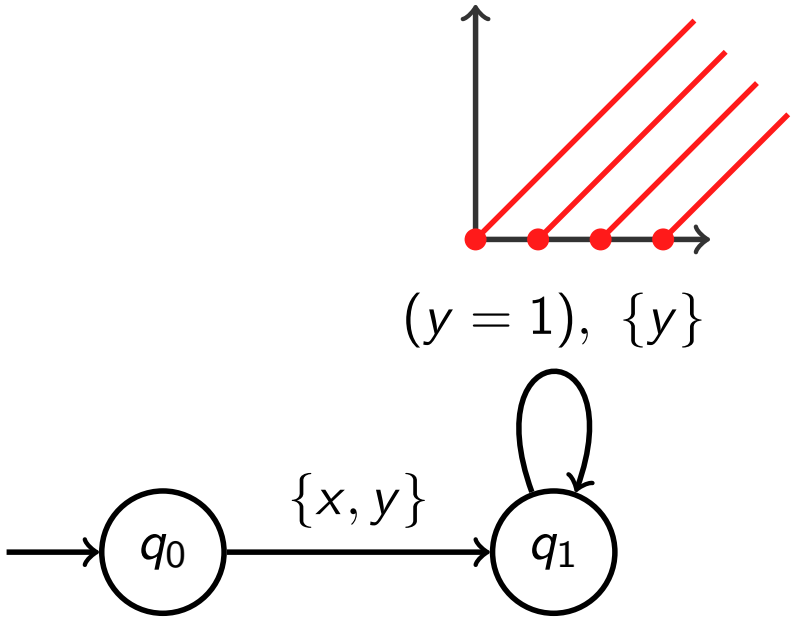
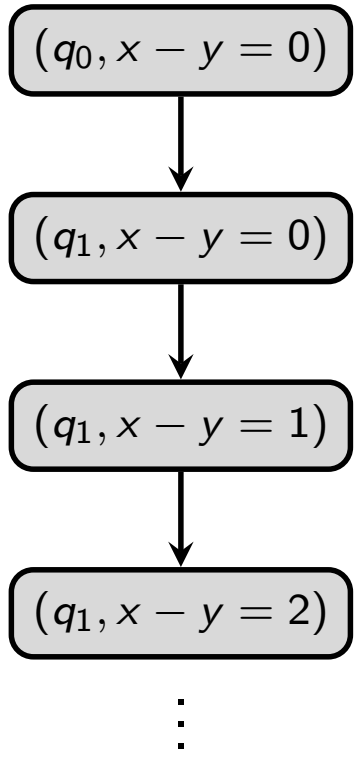
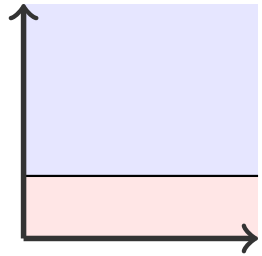




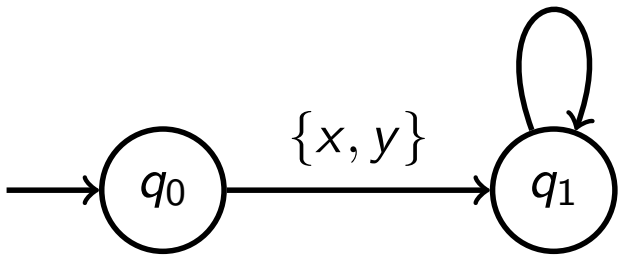
Abstract zone to its **region closure**

$M(x) = -\infty$

$M(y) = 1$



$(y = 1),\ \{y\}$



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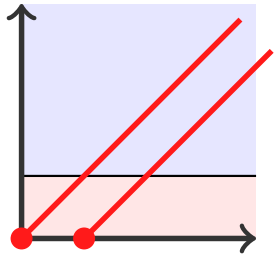

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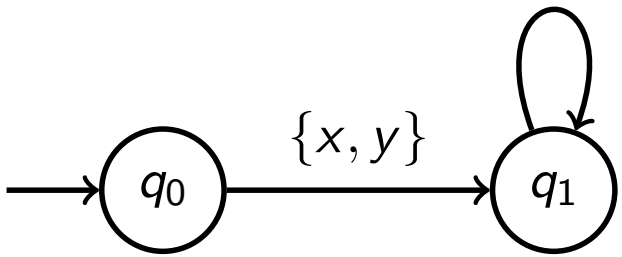

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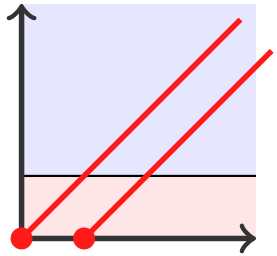

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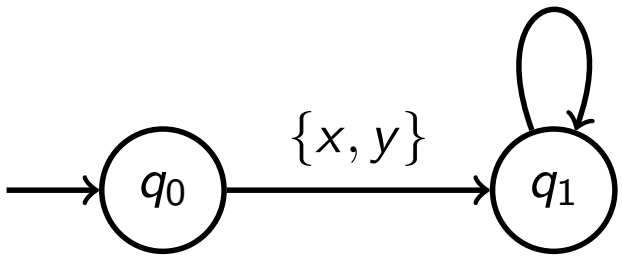

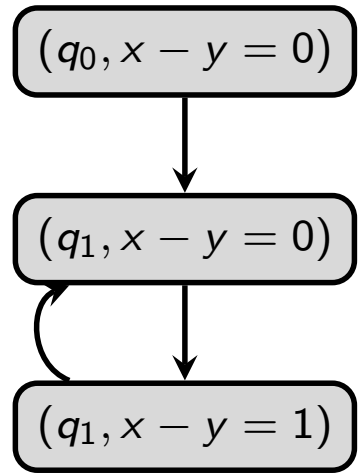

$$x - y = 1 \ \subseteq\ \mathsf{Closure}_M(x - y = 0)$$

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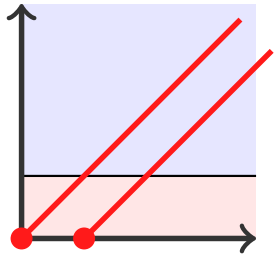

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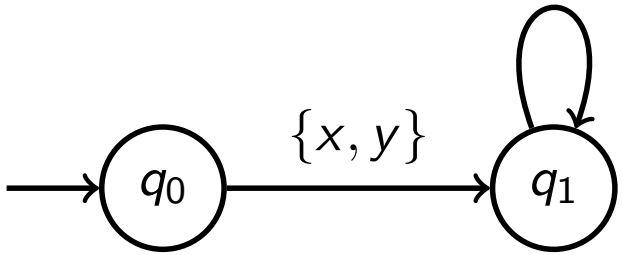

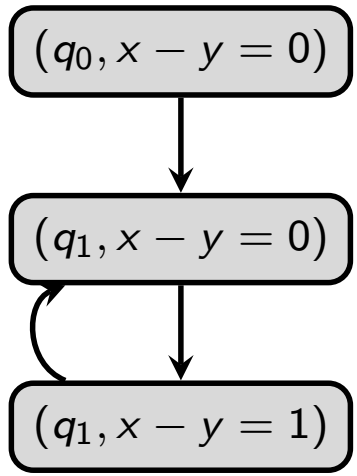

$$x - y = 1 \ \subseteq\ \text{Closure}_M(x - y = 0)$$

Using Closure

1. $Z \subseteq \text{Closure}_M(Z')$ can be done **efficiently** [HKSW11]
2. Given M, Closure_M is **optimal** [HSW12]

Reachability algorithm:

- Compute zones
- Use Closure$_M$ for termination
- Given M, Closure$_M$ is optimal

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- Use Closure_M for termination
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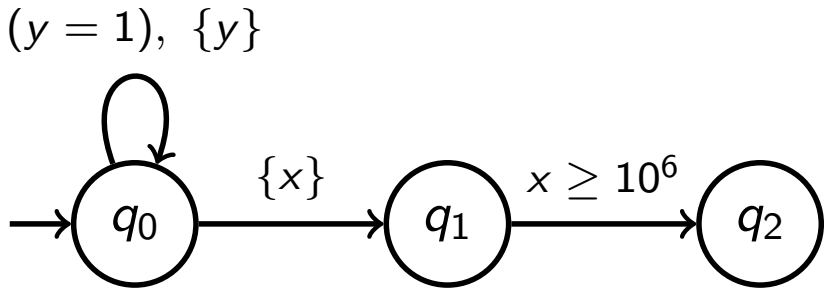
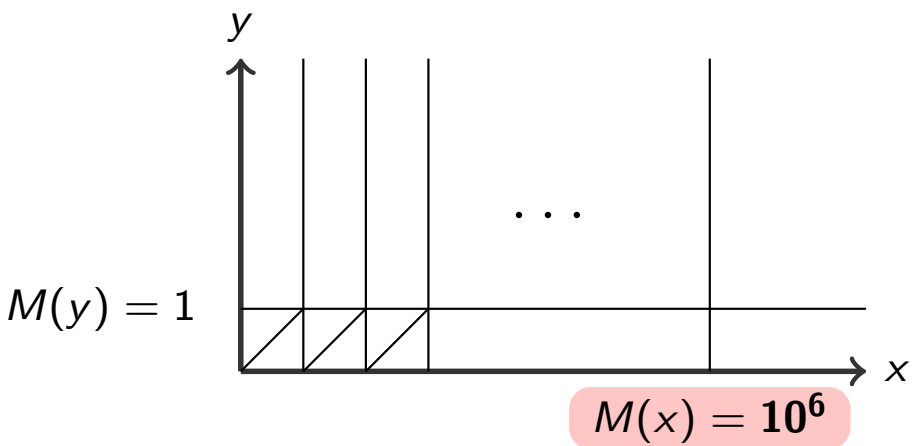
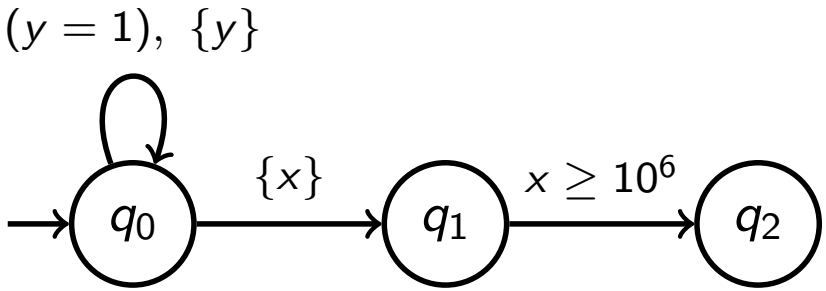
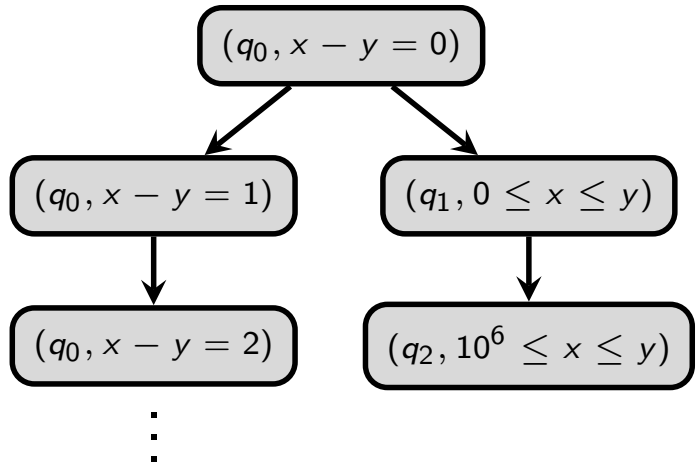
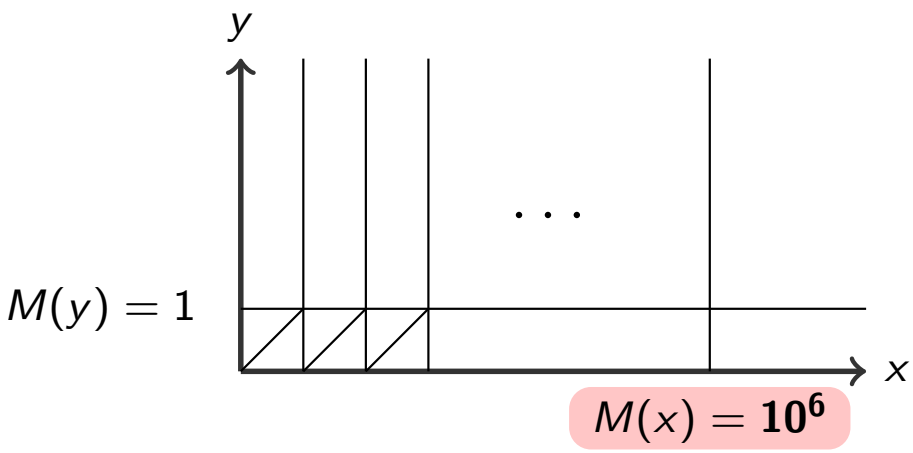
Coming next: get **better** M bounds!

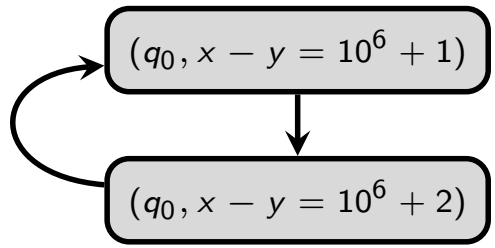

$(y = 1),\ \{y\}$
$\{x\}$
$x \geq 10^6$
q_0
q_1
q_2


y
$M(y) = 1$
x
$M(x) = \mathbf{10^6}$





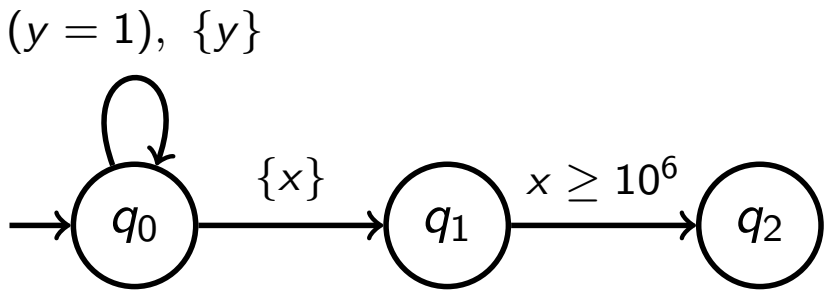




More than 10^6 nodes unnecessary

Static analysis [BBFL03]

Key idea: Bounds for every q of the automaton



$M_0(x) = -\infty$ $\quad M_1(x) = 10^6$

$M_0(y) = 1$ $\quad M_1(y) = -\infty$

Static analysis [BBFL03]

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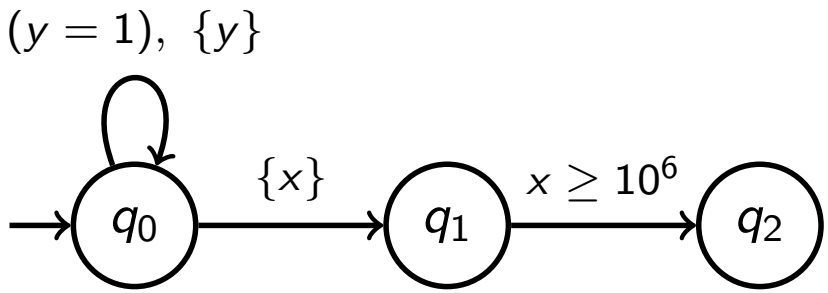

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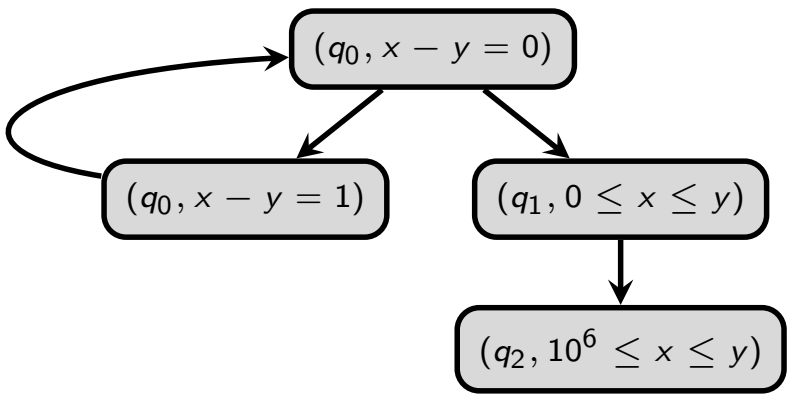
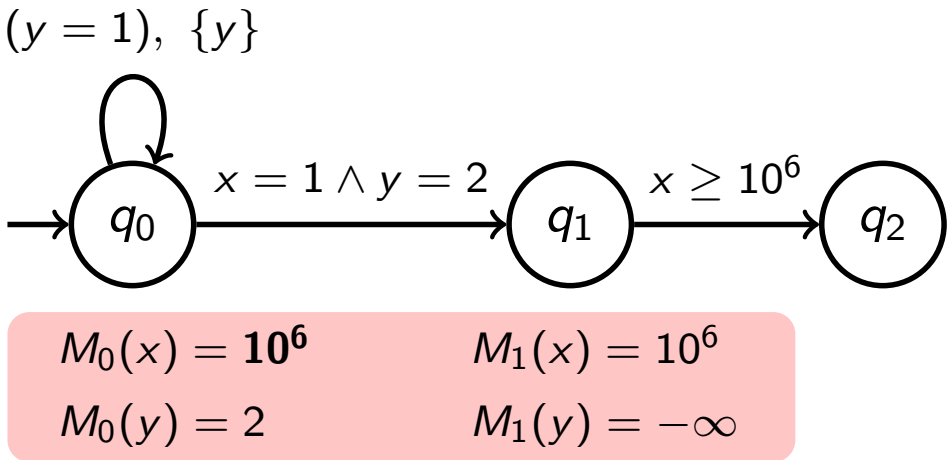
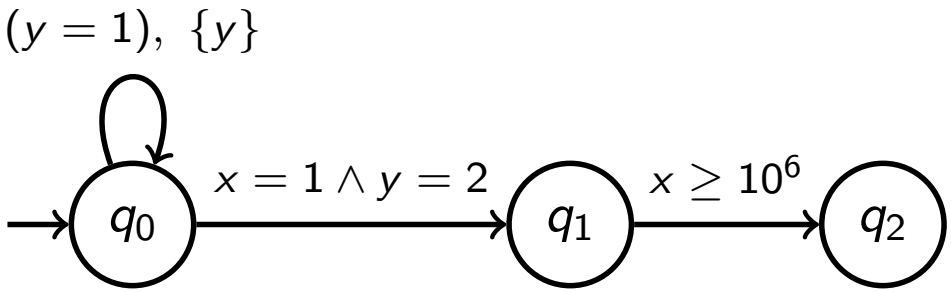


However...



Static analysis gives **more than** 10^6 nodes in the zone graph

However...



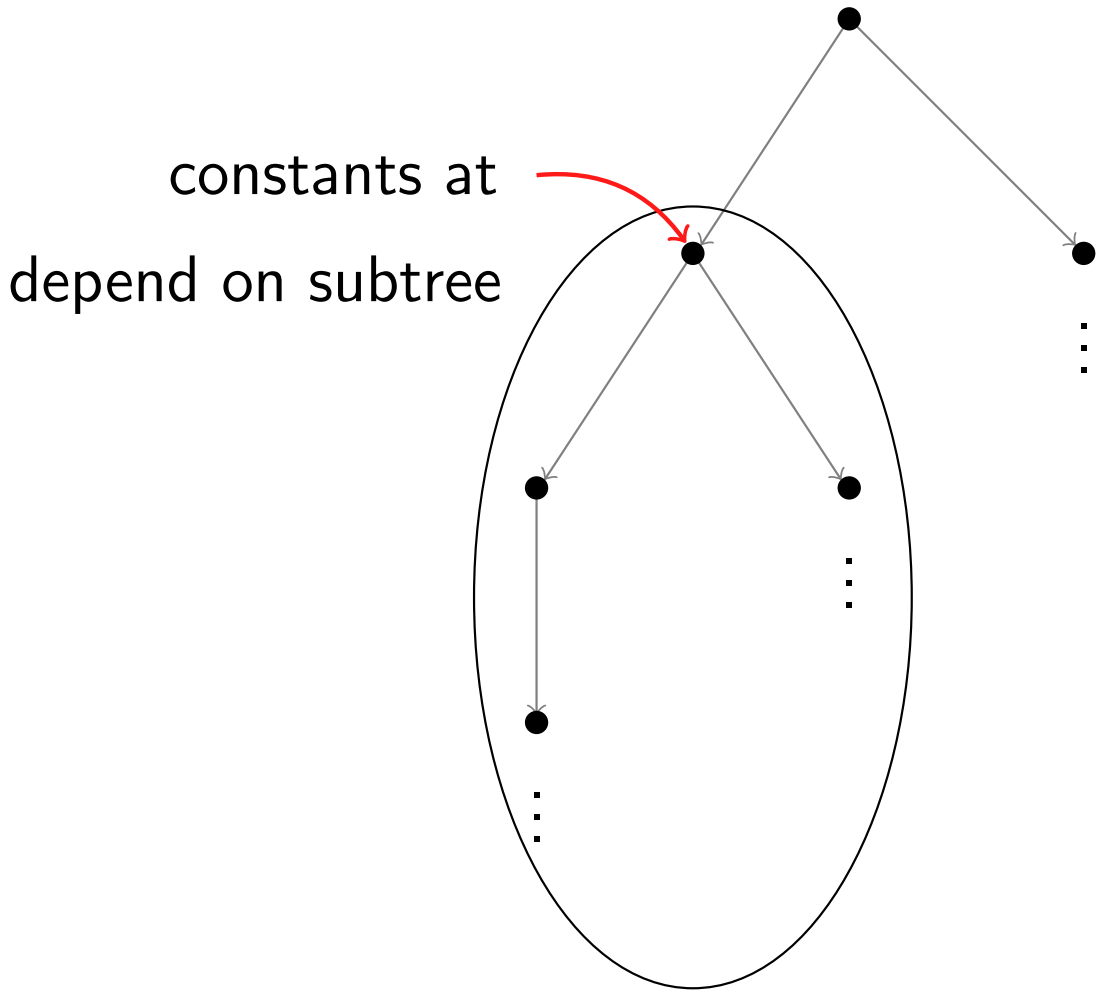
$M_0(x) = \mathbf{10^6}$	$M_1(x) = 10^6$
$M_0(y) = 2$	$M_1(y) = -\infty$

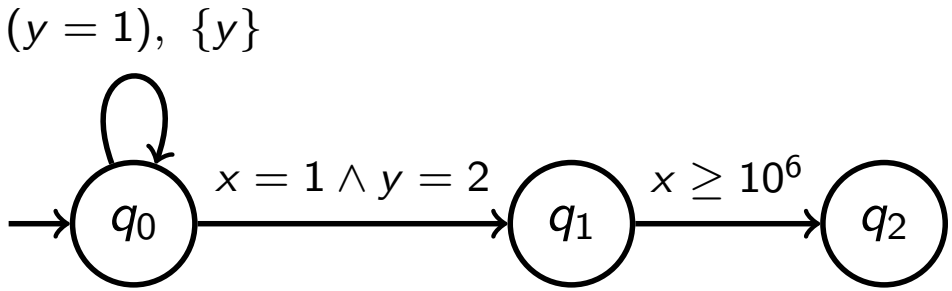
Static analysis gives **more than** 10^6 nodes in the zone graph

Need to look at **semantics**

On-the-fly bounds [HKSW11]

Key idea: Bounds for every (q, Z) of the zone graph

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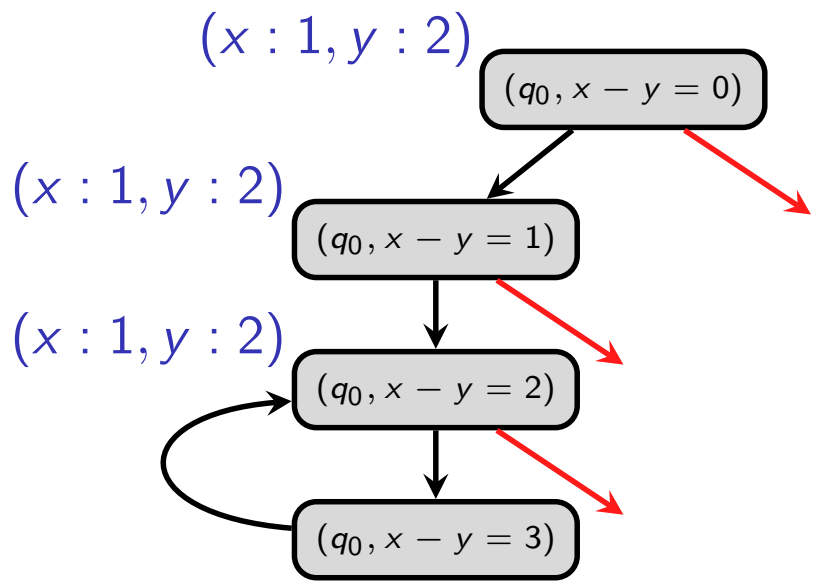
$(x : 1, y : 2)$
$(q_0, x - y = 0)$
$(x : 1, y : 2)$
$(q_0, x - y = 1)$
$(x : 1, y : 2)$
$(q_0, x - y = 2)$
$(q_0, x - y = 3)$

Two ways of getting bounds:

- **Static** analysis (bounds for every q)
- **On-the-fly** propagation (bounds for every (q, Z))

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- **Static** analysis (bounds for every q)
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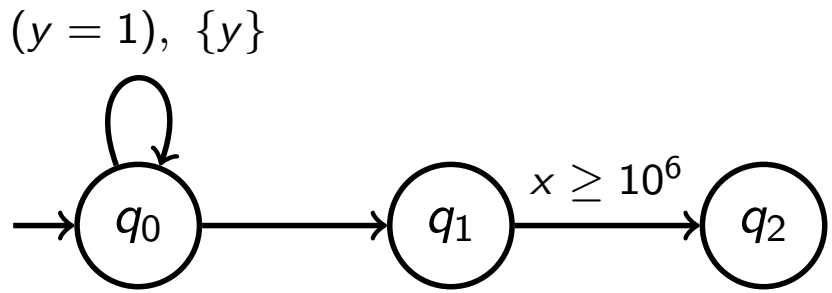
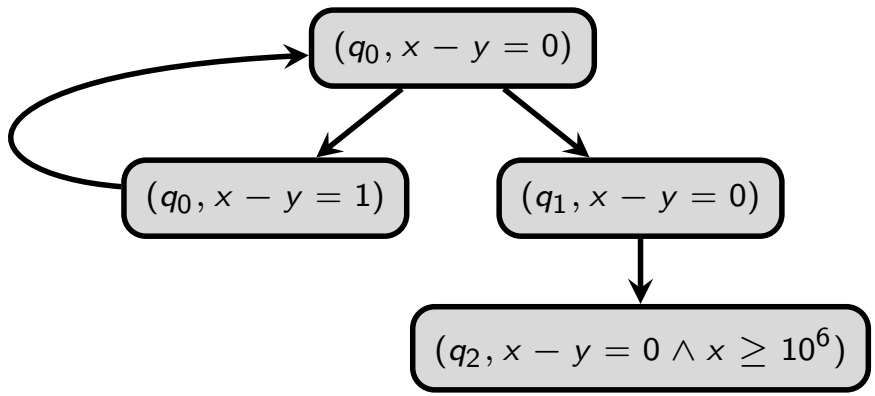
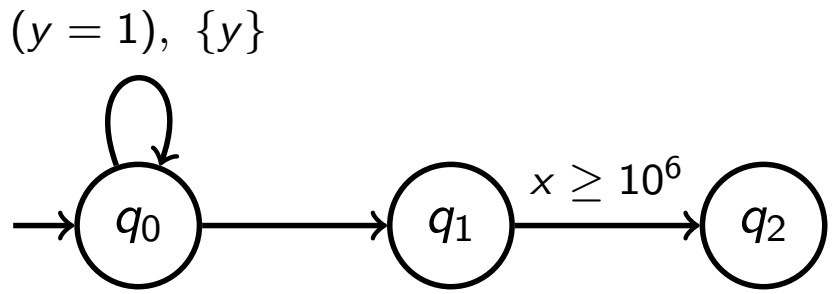
Coming next: Better bounds by exploiting **more semantics**

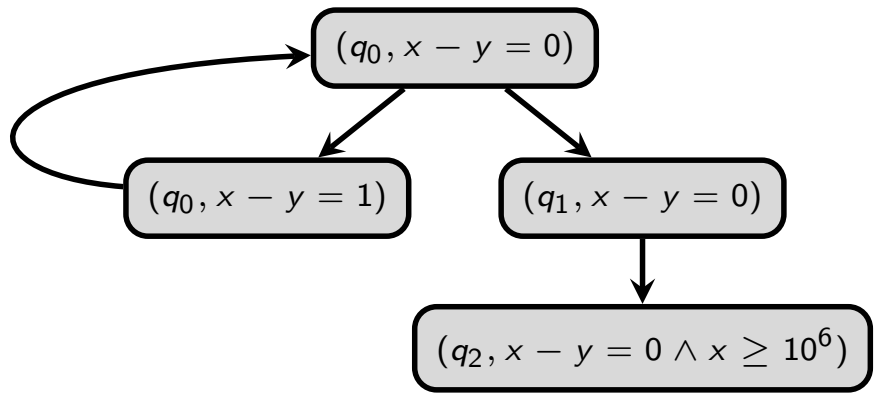
Observation 1:

If **all edges are enabled** in the zone graph, then we **don't need** bounds at all

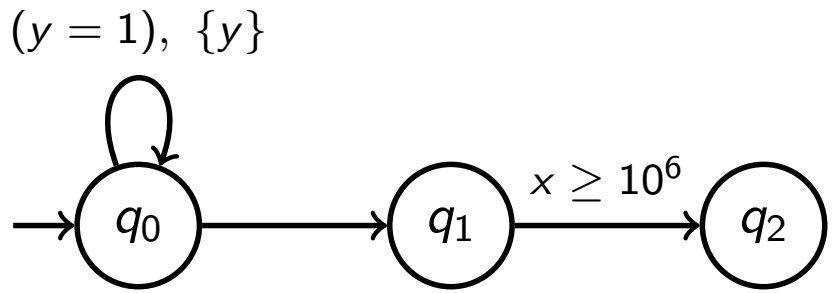

$(y = 1),\ \{y\}$
q_0
q_1
$x \geq 10^6$
q_2

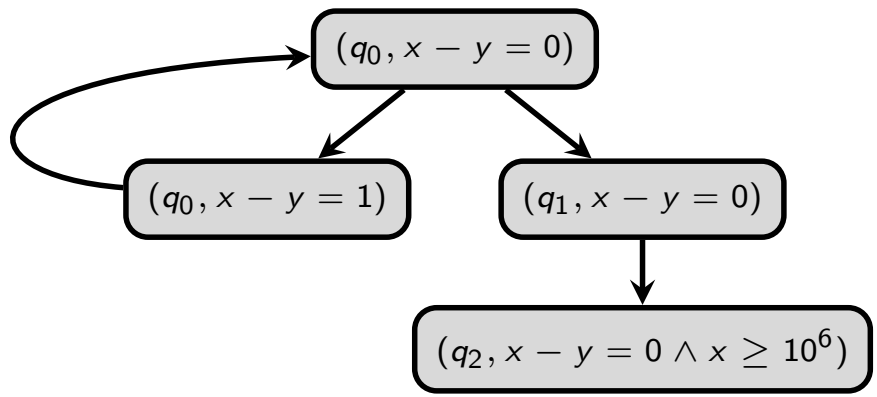

$(q_0, x - y = 0)$
$(q_0, x - y = 1)$
$(q_1, x - y = 0)$
$(q_2, x - y = 0 \wedge x \geq 10^6)$





Otf-propagation would have given $\sim 10^6$ nodes



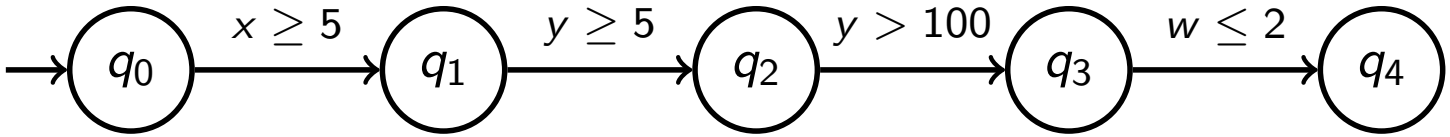


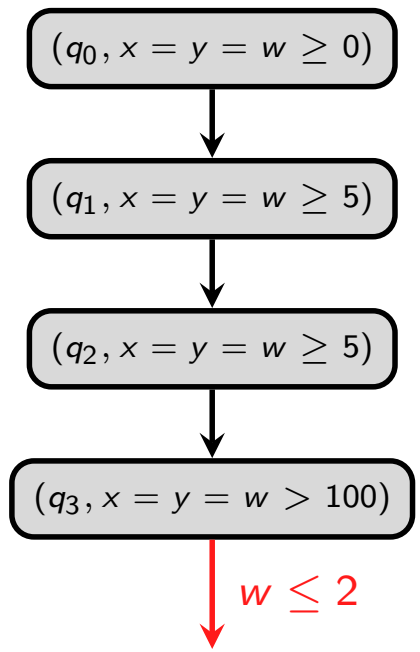
Otf-propagation would have given $\sim 10^6$ nodes

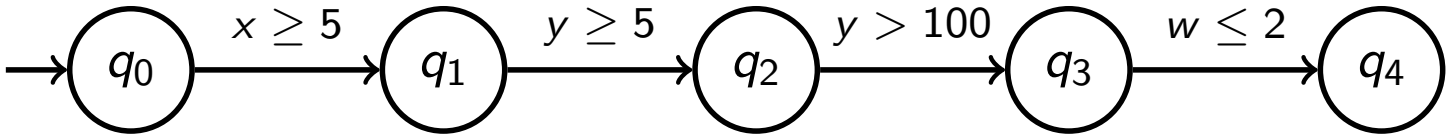
Trigger bounds propagation only when a **disabled edge** is seen

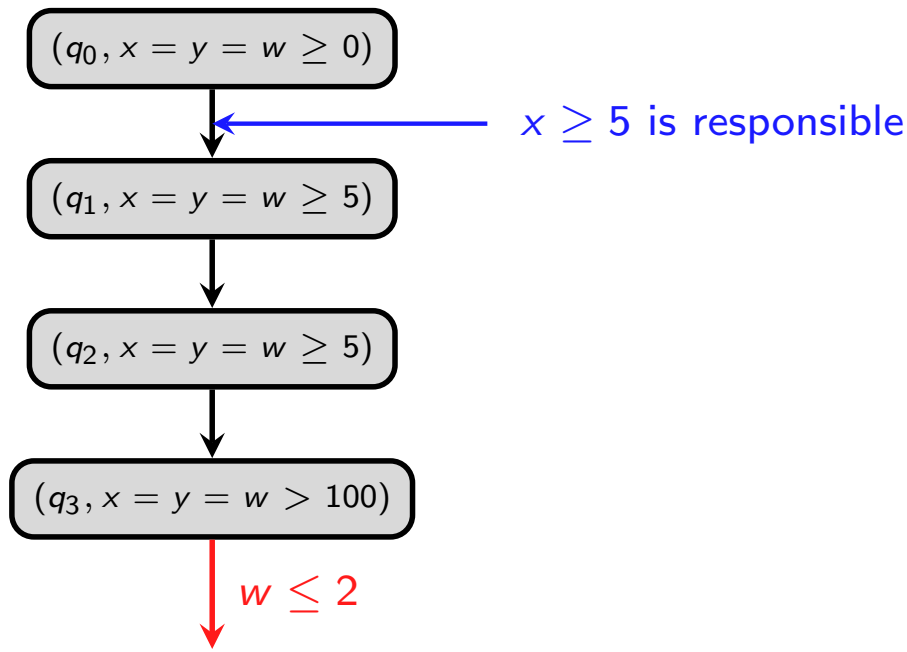
Observation 2:

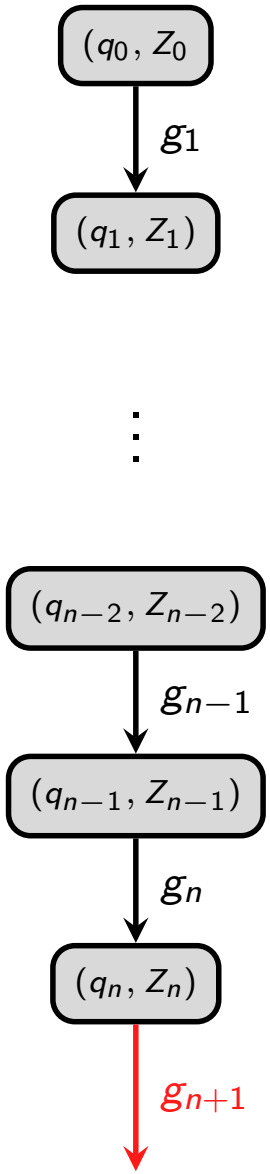
If **some edge is disabled** in the zone graph, it is enough to consider only the **guards that were responsible** for the edge to be disabled

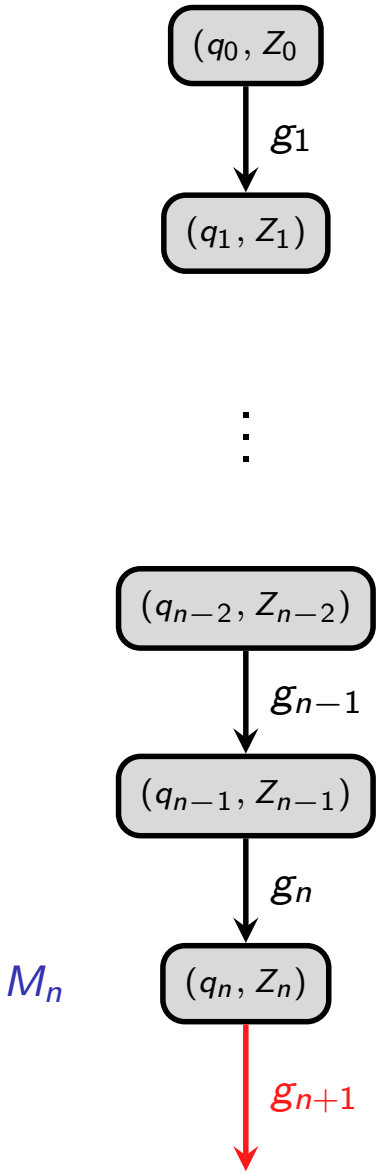

q_0
$x \geq 5$
q_1
$y \geq 5$
q_2
$y > 100$
q_3
$w \leq 2$
q_4


$(q_0, x = y = w \geq 0)$
$(q_1, x = y = w \geq 5)$
$(q_2, x = y = w \geq 5)$
$(q_3, x = y = w > 100)$
$w \leq 2$


q_0
$x \geq 5$
q_1
$y \geq 5$
q_2
$y > 100$
q_3
$w \leq 2$
q_4


$(q_0, x = y = w \geq 0)$
$x \geq 5$ is responsible
$(q_1, x = y = w \geq 5)$
$(q_2, x = y = w \geq 5)$
$(q_3, x = y = w > 100)$
$w \leq 2$

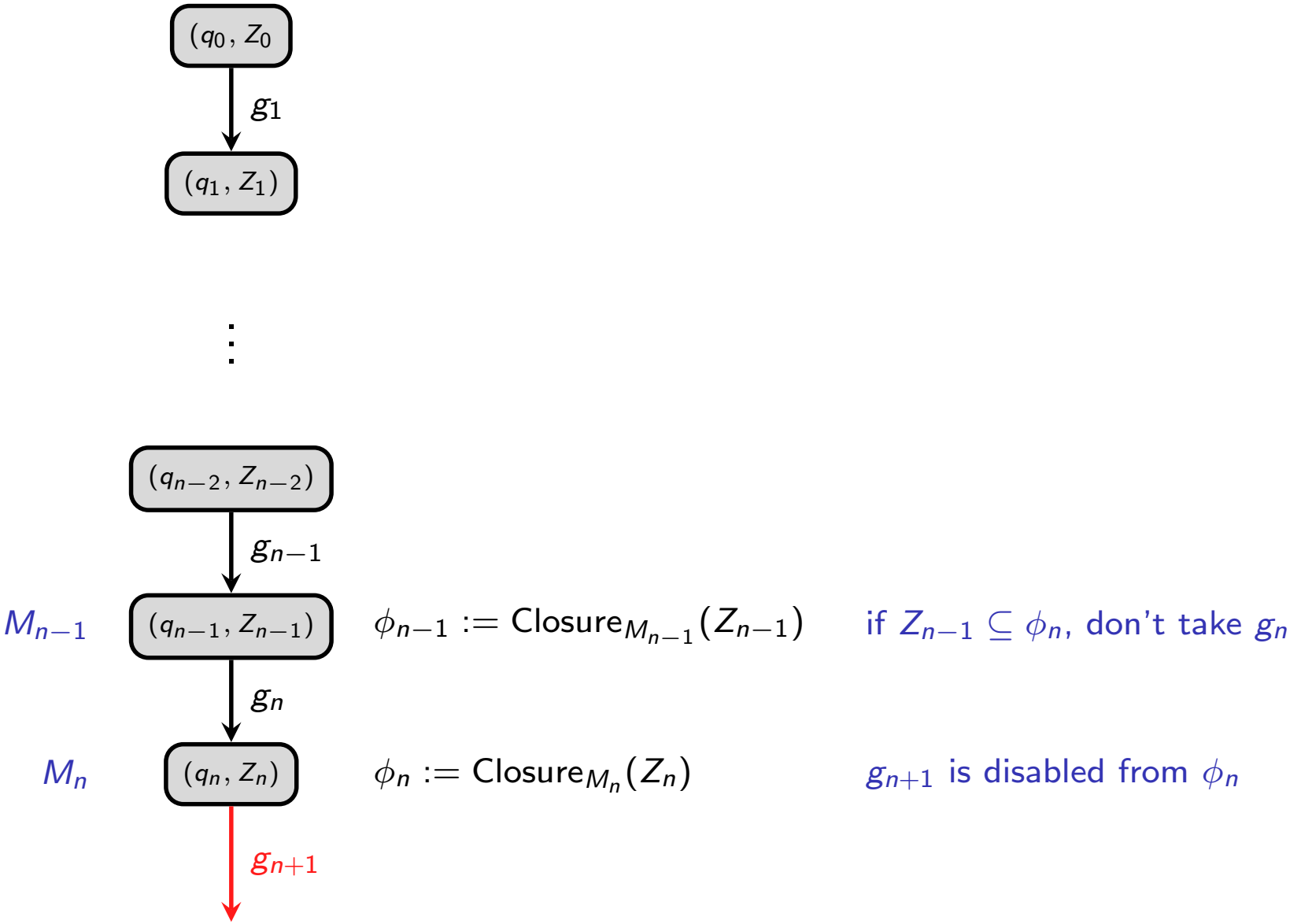

(q_0, Z_0)
g_1
(q_1, Z_1)
(q_{n-2}, Z_{n-2})
g_{n-1}
(q_{n-1}, Z_{n-1})
g_n
(q_n, Z_n)
g_{n+1}

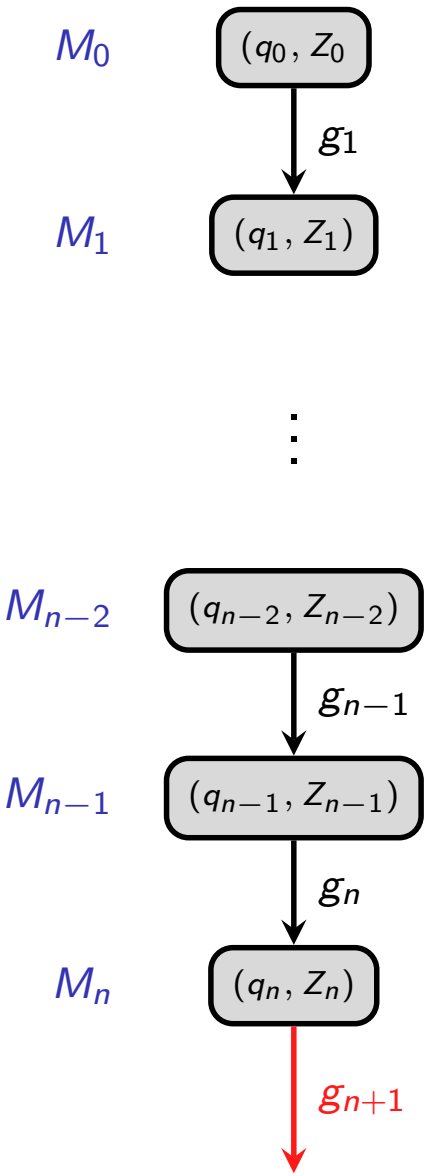


M_n

$\phi_n := \mathsf{Closure}_{M_n}(Z_n)$

g_{n+1} is disabled from ϕ_n


(q_0, Z_0)
g_1
(q_1, Z_1)
(q_{n-2}, Z_{n-2})
g_{n-1}
M_{n-1}
(q_{n-1}, Z_{n-1})
$\phi_{n-1} := \text{Closure}_{M_{n-1}}(Z_{n-1})$
if $Z_{n-1} \subseteq \phi_n$, don't take g_n
g_n
M_n
(q_n, Z_n)
$\phi_n := \text{Closure}_{M_n}(Z_n)$
g_{n+1} is disabled from ϕ_n
g_{n+1}



$\phi_1 := \text{Closure}_{M_0}(Z_0)$ if $Z_0 \subseteq \phi_1$, don't take g_1

$\phi_1 := \text{Closure}_{M_1}(Z_1)$ if $Z_1 \subseteq \phi_2$, don't take g_2

$\phi_{n-1} := \text{Closure}_{M_{n-2}}(Z_{n-2})$ if $Z_{n-2} \subseteq \phi_{n-1}$, don't take g_{n-1}

$\phi_{n-1} := \text{Closure}_{M_{n-1}}(Z_{n-1})$ if $Z_{n-1} \subseteq \phi_n$, don't take g_n

$\phi_n := \text{Closure}_{M_n}(Z_n)$ g_{n+1} is disabled from ϕ_n

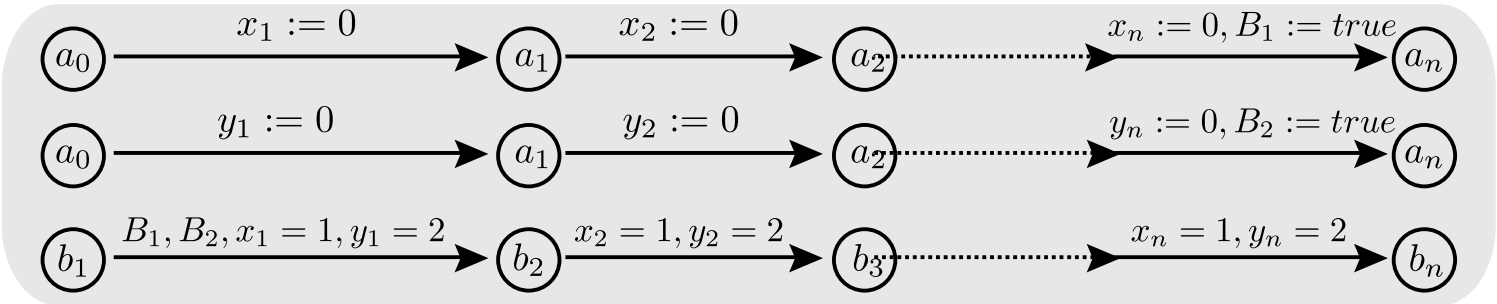
Lazy propagation

A new **efficient** propagation algorithm based on relation between **successor** zones

Lazy abstractions for timed automata (CoRR abs/1301.3127)

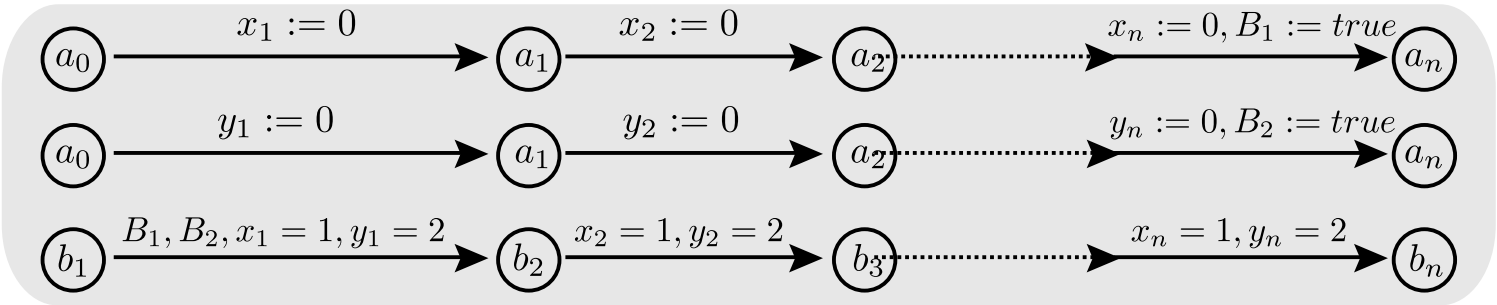
Exponential gain

$\mathcal{D}'_n$



Exponential gain

$\mathcal{D}'_n$



- **Lazy:** gives constants only for some pair (x_i, y_i) in any path **(quadratic zone graph)**
- **Static:** gives constants for all clocks **(exponential)**
- **Otf:** gives constants for k clocks depending on order of exploration **(exponential)**

Experiments

Model	nb. of clocks	UPPAAL (-C) nodes	UPPAAL (-C) sec.	static nodes	static sec.	otf nodes	otf sec.	lazy nodes	lazy sec.
CSMA/CD 10	11	120845	1.9						
CSMA/CD 11	12	311310	5.4						
CSMA/CD 12	13	786447	14.8						
FDDI 50	151	12605	52.9						
FDDI 70	211								
FDDI 140	421								
Fischer 9	9	135485	2.4						
Fischer 10	10	447598	10.1						
Fischer 11	11	1464971	40.4						
Stari 2	7	7870	0.1						
Stari 3	10	136632	1.7						
Stari 4	13	1323193	26.2						

- **UPPAAL (-C)** shows results from UPPAAL tool which uses static bounds
- **static** is our implementation of UPPAAL's algo
- Time out (150s), Memory out (1Gb)

Experiments

Model	nb. of clocks	UPPAAL (-C) nodes	UPPAAL (-C) sec.	static nodes	static sec.	otf nodes	otf sec.	lazy nodes	lazy sec.
CSMA/CD 10	11	120845	1.9	120844	6.3				
CSMA/CD 11	12	311310	5.4	311309	16.8				
CSMA/CD 12	13	786447	14.8	786446	44.0				
FDDI 50	151	12605	52.9	12606	29.4				
FDDI 70	211								
FDDI 140	421								
Fischer 9	9	135485	2.4	135485	8.9				
Fischer 10	10	447598	10.1	447598	34.0				
Fischer 11	11	1464971	40.4	1464971	126.8				
Stari 2	7	7870	0.1	6993	0.4				
Stari 3	10	136632	1.7	113958	9.4				
Stari 4	13	1323193	26.2	983593	109.0				

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Experiments

Model	nb. of clocks	UPPAAL (-C) nodes	UPPAAL (-C) sec.	static nodes	static sec.	otf nodes	otf sec.	lazy nodes	lazy sec.
CSMA/CD 10	11	120845	1.9	120844	6.3				
CSMA/CD 11	12	311310	5.4	311309	16.8				
CSMA/CD 12	13	786447	14.8	786446	44.0				
FDDI 50	151	12605	52.9	12606	29.4	5448	14.7	401	0.8
FDDI 70	211							561	2.7
FDDI 140	421							1121	37.6
Fischer 9	9	135485	2.4	135485	8.9				
Fischer 10	10	447598	10.1	447598	34.0				
Fischer 11	11	1464971	40.4	1464971	126.8				
Stari 2	7	7870	0.1	6993	0.4	5779	0.4	5113	0.5
Stari 3	10	136632	1.7	113958	9.4	82182	8.2	53178	7.8
Stari 4	13	1323193	26.2	983593	109.0	602762	84.9	342801	65.7

- **UPPAAL (-C)** shows results from UPPAAL tool which uses static bounds
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Experiments

Model	nb. of clocks	UPPAAL (-C) nodes	UPPAAL (-C) sec.	static nodes	static sec.	otf nodes	otf sec.	lazy nodes	lazy sec.
CSMA/CD 10	11	120845	1.9	120844	6.3	78604	6.1	74324	6.1
CSMA/CD 11	12	311310	5.4	311309	16.8	198669	16.1	188315	15.9
CSMA/CD 12	13	786447	14.8	786446	44.0	493582	41.8	469027	40.9
FDDI 50	151	12605	52.9	12606	29.4	5448	14.7	401	0.8
FDDI 70	211							561	2.7
FDDI 140	421							1121	37.6
Fischer 9	9	135485	2.4	135485	8.9	135485	11.4	135485	24.7
Fischer 10	10	447598	10.1	447598	34.0	447598	42.8	447598	98.1
Fischer 11	11	1464971	40.4	1464971	126.8				
Stari 2	7	7870	0.1	6993	0.4	5779	0.4	5113	0.5
Stari 3	10	136632	1.7	113958	9.4	82182	8.2	53178	7.8
Stari 4	13	1323193	26.2	983593	109.0	602762	84.9	342801	65.7

- **UPPAAL (-C)** shows results from UPPAAL tool which uses static bounds
- **static** is our implementation of UPPAAL's algo
- Time out (150s), Memory out (1Gb)

Summary and future work

- A **new algorithm** for obtaining abstraction parameters
  - works the same way for LU-bounds and $\mathfrak{a}_{\preccurlyeq_{LU}}$ abstraction
- Theoretically, **exponential** gain possible
- Practical gains understood by **experiments**

- Better **data-structures** for zones
- Abstractions for **discrete** component
- Probabilistic systems, (rectangular) hybrid systems

References I

R. Alur and D.L. Dill.

A theory of timed automata.
Theoretical Computer Science, 126(2):183–235, 1994.

G. Behrmann, P. Bouyer, E. Fleury, and K. G. Larsen.

Static guard analysis in timed automata verification.
In *TACAS'03*, volume 2619 of *LNCS*, pages 254–270. Springer, 2003.

C. Daws and S. Tripakis.

Model checking of real-time reachability properties using abstractions.
In *TACAS'98*, volume 1384 of *LNCS*, pages 313–329. Springer, 1998.

F. Herbreteau, D. Kini, B. Srivathsan, and I. Walukiewicz.

Using non-convex approximations for efficient analysis of timed automata.
In *Proceedings of FSTTCS*, volume 13 of *LIPIcs*, pages 78–89. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2011.

F. Herbreteau, B. Srivathsan, and I. Walukiewicz.

Better abstractions for timed automata.
In *LICS*, 2012.